An analysis of students' difficulties in conjecturing process of block paving problems

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Abstract. This study aimed to describe students' difficulties in the conjecturing process of block paving problems. The subjects used in study were 32 of 8th grade students of Junior High School. The data were collected using Block Paving Problems (BPP) and interviews. Data collection procedures consisted of two stages. The first stage was the researchers had the student solved BPP while think aloud. The second stage is task-based interview to explore information needed to confirm from the results of think aloud. The result of study revealed that 1) difficulty in interpreting the pattern of n, 2) The difficulty in finding and predicting the next pattern based on regularity, 3) The difficulty in finding and predicting the next pattern 4) The difficulty in Forming Pattern of n.

1. Introduction

Basically the process of learning mathematics is closely related to the formation and use of plying / reasoning abilities. Students will readily accept or process information science if there is a scheme of thinking, so that when they are faced with a mathematical problem, it is easy to place, manage, arrange and use a logical flow. Peretz in his study states that what students first need in the process of reasoning is to feel "need" to reason, to develop their reasoning, or habitual thinking [1, 25].

One way to develop students' reasoning abilities is through conjecturing. One standard of reasoning ranging from pre-kindergarten to high school is to create and investigate mathematical conjectures [2]. Creating and investigating mathematical conjecture is a conjecturing process.

The process of conjecturing is a mental activity in building conjectures based on the knowledge already possessed [8]. Mental activity in constructing conjectures is a process that occurs in the mind (thought process) that appears through students' behavior in problem solving. [9] states that problem solving and conjecturing are an important and interconnected part of mathematical activity. It further explained that problem solving involves the invention, conjecturing is the main route to the invention [2].

Research on the process of conjecturing in problem solving has been widely practiced ([9], [10], [11], [6], [25]). Cañadas, et al., (2007) describes the familiar types and stages of conjecturing in mathematics education. in solving the problem of open classical analogy through conjecturing,

students seek perceptual, transitional, and relational similarities [10]. Designing tasks involving elementary students in Conjecturing the activities and proofs [11]

The process of conjecturing in problem solving generalization patterns consists of local and global conjecturing processes. The global process of conjecturing in solving the problem of pattern generalization takes place at the stage of action, where students observe and organize cases as a whole to build conjecture [7,8]. While the local conjecturing process in solving the problem of pattern generalization occurs at the stage of action, where students observe and organize cases separately as a basis for constructing conjectures. Among these studies, It has not revealed the difficulties experienced by students in the conjecturing process.

Generalizing patterns is an important aspect of mathematics in each topic and is something that is highlighted in teaching, almost at all levels [3,4,5]. Generalizing patterns is the activity of generating general rules based on specific examples ([6], [7]), whereas according to [4] generalizations should be at the core of mathematical activities in schools. In general it is inseparable from pattern analysis.

Patterns are ideas that underlie mathematical thinking. According to [15] pattern analysis, describing patterns, and their properties are one of the goals of mathematics. [5] states almost all mathematics is based on patterns and structures. One of the basic competencies of SMP / MTs mathematics is to understand patterns and use them to guess and make generalizations or conclusions [16].

Generalization in school math cannot ignore the child's psychology of children or students generalizing in their ways. [17 states that children not only use notations/symbols but also must present and give mathematical reasons, make conclusions and generalizations in their own way. [2] describes one of the standards that should be studied and mastered by students of grade 6 to grade 8 is to understand patterns, relationships, and functions. In understanding the pattern, students are required to represent, analyze, and generalize the pattern with tables, graphs, diagrams, symbols and others.

[18] mentions that mathematical patterns can be described as predictable regularities, usually involving numerical, spatial, or logical relationships. [19] states that patterns can be graphic, numeric, verbal and algebraic. Furthermore, this study uses a graphic-shaped pattern (pictorial), because the pictorial pattern allows one to observe in different ways [6]

Associated with patterns, many mathematicians claim that mathematics is called "the science of pattern" [20, 21, 25]. This perspective highlights the existence of patterns in all areas of mathematics. In particular, the pattern is seen by some researchers as a way of approaching algebra because it is a fundamental step to build a generalization which is the essence of mathematics [22]).

Furthermore, [23] explains three types of generalizations of patterns, namely (1) factual generalization, (2) contextual generalization, and (3) symbolic generalizations. The type of factual generalization is a general type based on known facts. The factual trait affirms the idea that this generalization takes place at a basic level of generalization in which the universal speech does not go beyond a particular sequence or image, such as a figure to 1000, an image to 1000000, and so on. The types of factual generalization at the level of action (action) numerically and allows students to cope with certain cases. A contextual generalization type is a type of generalization based on a problem context and limited to a particular object. A symbolic generalization type is a type of generalization associated with an algebraic object or symbol that is not limited to a particular object.

The type of generalization developed by Radford, implicitly illustrates that there are children who experience great-natures. For example a child who does a factual generalization actually has difficulty to contextually or symbolically generalize. Therefore, it is important to understand more about the difficulties experienced by students in generalizing the pattern.

For a teacher, it is necessary to know the difficulties experienced by students in the learning process. Difficulties can be seen from the mistakes made in solving the problem on the given problem. The search for errors is an effort to be done to improve the learning achievement of learners' mathematics. Problems that are not immediately addressed will result in a lack of student understanding on later higher mathematical topics. This resulted in the accumulation of student disability because of the complexity of his problems in mastering mathematical concepts.

Difficulties are often reflected errors in problems solving. Of the 62 students who did the proses conjecturing in solving the problem of generalization pattern only 43.5% of students successfully completed, while 56.5% of students did not succeed. [13, 25] The failure of the students to solve the problem indicates the difficulties experienced by the students in solving the problem.

Students who have difficulty in the process of conjecturing, need more attention, because if not immediately addressed, it will impact on other problems. To overcome student difficulties, it is necessary to trace the source of the error. In this study, it can be done with cognitive map (cognitive map). Explaining that the cognitive map shows the direction of thinking in such a way that can be as a guide to go further. The cognitive map differs from the concept map. The concept map shows the concept hierarchy relationship, while the cognitive map describes one's thinking flow in constructing or solving problems. Therefore, the cognitive map does not indicate hierarchy, but rather describes the connection between knowledge, problems, procedures, and concepts of thought processes. Student difficulties in generalizing patterns in the process of conjecturing students can be photographed using cognitive maps and can be traced the location of the errors.

2. Methodology

Subjects involved in this study were 35 students of grade VIII from MTS Surya Buana Malang and student of Junior High School 1 Ponorogo. The data were collected using Block Paving Problem (BPP) and interviews. Data collection procedures consisted of two stages. The first stage was the researchers had the student solved BPP while thinking aloud. The second stage was task-based interview to explore information needed to confirm from the results of think alouds.

This research was a qualitative research embraced an explorative descriptive approach. Data analysis were conducted by; (1) transcribing data obtained from interviews and think aloud, (2) condensing data, (3) data encoding, (4) describing students' difficulties in conjecturing process, and (5) conclusions. The BPP is presented in Figure 1.

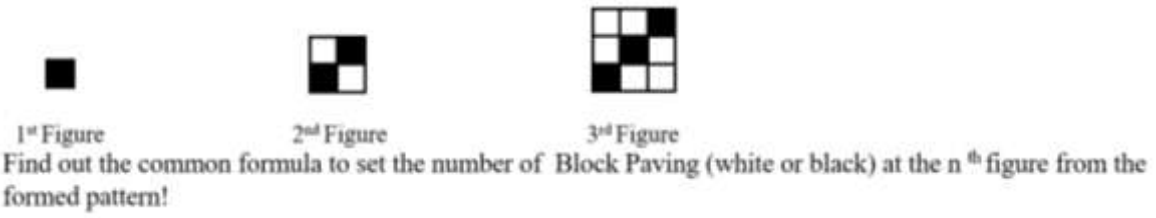


Figure 1. Paving Block Problem (BPP)

3. Results And Discussion

This section will describe the students' difficulties in the conjecturing process based on the stages of the conjecturing process undertaken by each subject. Stages of the conjecturing process consist of observing cases, organizing cases, searching and predicting patterns, formulating conjectures, validating conjectures, generalizing conjectures and justifying conjectures.

The thirty-two (32) subjects used in this study were low-ability students and could not complete BPPs provided. Of these 32, all perform the stages of observing the case and having difficulty 7, in organizing the case of 17 subjects performing and 11 experiencing difficulties, at the stage of searching and predicting patterns 12 do and having difficulty 7, in the stage of formulating conjecture 12 people do and who had difficulty 8 people, in the stage of validating the conjecture 7 people did and no one had difficulty. Being in the stage of generalizing and justifying the conjecture no one does. Because of difficulty in formulating conjecture. The following data of students who have difficulty are presented in the following table.

Table 1. Data Subject Having Difficulty

Stage of Conjecturing Process	Number of subjects who carry it out	Number of Subjects are Having Difficulty

Observe the case	32	7
Organize the Case	19	11
Finding and Predicting Patterns	12	7
Formulating Conjecture	7	7
Validating Conjecture	7	-
Generalizing conjecture	-	-
Justify conjecture	-	-

The results of data analysis from 32 subjects shows obtained difficulties experienced by students in the conjecturing process is as follows

4. Results and Discussion

*4.1 Difficulty to interpret the **n** pattern*

The first stage is the researcher asks each student to solve a given problem while thinking alouds (voicing what is thought). In the second stage, students are asked to solve the same problem as a

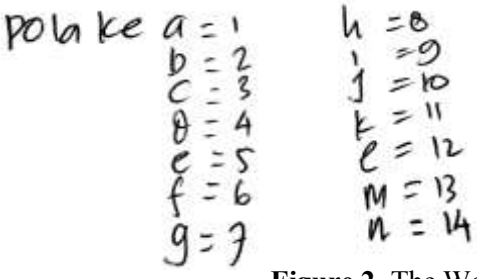
Observing the case, the subject S1 performs the activities in understanding BPP by calculating the 1st pattern of 1, the 2nd pattern is 4, the 3rd pattern is 9 and understanding the BPP question. Understanding the question, the subject S1 has difficulty in interpreting the question in determining plenty of paving blocks on the pattern of the N. the difficulty is seen at the time of S1 mention the pattern of the n-pattern of how ? The following S1 subject statement is based on the excerpt of think aloud data.

S1: 1st black pattern 1, 2nd pattern black 2 white 2, 3rd pattern black 3 white 6. Determine the number of black and white paving blocks on the nth pattern. The pattern of the n, the pattern to how ya? What is the 10th pattern, the 25th pattern

After understanding the BPP subject S1 continues to count the number of black and white squares on each pattern. Due to difficulties in understanding the question on the nth subject pattern, the subject stops to complete the BPP. Here are excerpts of interviews of researchers and subject S1.

P : Brother, while you think aloud do you mentioned the pattern of n, the number of pattern ya? What does he mean brother
S1 : I'm still confused the pattern of N, and at what number is it sir.
P : I confusd what does it ment?
S1 : I am confused sir, the nth pattern is whether it is the 10th pattern or the 25th pattern
P : oo… so why do not you continue solving the problem.
S1 : I am confused to get it done. Sorry

The difficulty in interpreting the N pattern is seen in the subject S2 which states that the N pattern is the 14th pattern, since a = 1, b = 2, c = 3, ... n = 14. Here are the results of the S2 work in understanding the pattern of the n.



1st Pattern = a	8st Pattern = h
2nd Pattern = b	9nd Pattern = i
3rd Pattern = c	10rd Pattern = j
4th Pattern = d	11th Pattern = k
5th Pattern = e	12th Pattern = l
6th Pattern = f	13th Pattern = m
7th Pattern = g	14th Pattern = n

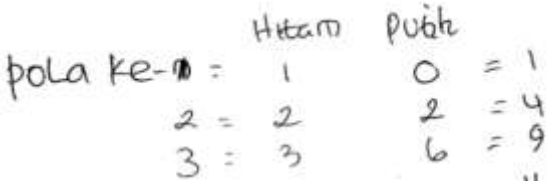
Figure 2. The Worked of S1

4.2 Difficulty In Finding Regular Pattern

Organizing phase of the case, S3 subject has difficulty in finding the regularity of the given pattern. It is seen when the subject confused in determining the relationship between pattern 1, pattern 2, and

pattern 3. Subjects who find regularity patterns tend to write down the number of black and white paving blocks of each pattern. [24, 25] the first thing that is involved in mathematical reasoning is the ability to determine patterns and structures to find order. The difficulty also occurs because the S3 subject does not organize the black and white paving block patterns to find the regularities that are formed. Activity organizing cases will make it easier to find the regularity of the pattern [9].

The subject of S7 also has difficulty in finding patterns of regularity. S7 is able to find that the 1st black pattern is 1 white 0, the 2nd black pattern is 2 white 2, the 3rd black pattern is 4 white 4 12. S7 has organized the case, but has not been able to find the regularity of the pattern. Here are the results of the S7 subject.



Black Pattern	White Pattern
1st Pattern = 1	0 = 1
2nd Pattern = 2	2 = 4
3rd Pattern = 3	6 = 9

Figure 3. The Worked of S7

P : brother, what do you think about this answer (while pointing to the S7)
S7 : I have found black and white paving, 1st pattern (1.0), 2nd pattern (2.2), 3rd pattern (3.9), but after that I am confused what to do, sir.

4.3 The Difficulty In Locating And Predicting The Pattern Further Is Based On Regularity

The stages in searching and predicting patterns, S3 subject felt difficulties for the next pattern. This is seen when S4 draws the 1st pattern of black 1, the 2nd pattern of the black 2, the 3rd pattern of the black 3 and proceeds to the 4th pattern of the black color 8, and the 5th pattern of the black color is 15. Mathematical patterns can be described as regularities of predictable objects, involving numerical, spatial or logical relationships [5]. The error performed by S4 occurs because it can not find the regularity of the given pattern. Here's the work of S3 in organizing cases.

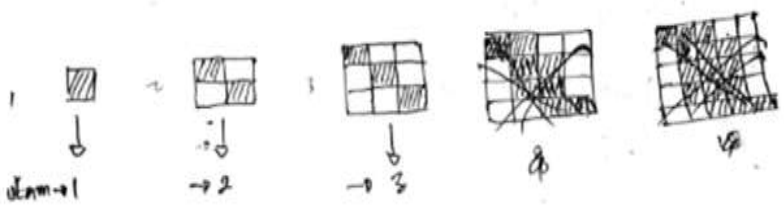


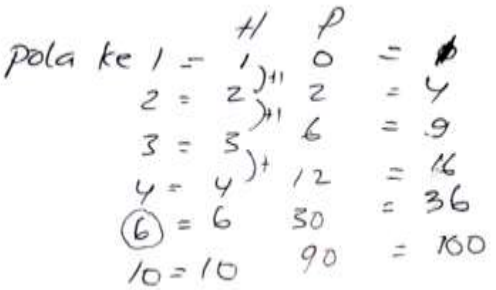
Figure 4. The Worked of S4

4.4 Difficulty in Forming Pattern of n

The stage of formulating n pattern, the subject conjecture S5 is difficult. It is seen when the S5 subject understands certain patterns, such as the 6th pattern of the number of white paving block 6 white 30, the 10th pattern of the black 10 white paving block 90. The S5 subject has difficulty in formulating the general shape or the nth pattern. The following is an excerption of a transcript of think aloud and the work of S2 in formulating the n pattern

P : How do you get this one (pointing to student work)
S : Oh, look, sir. For example the 6th image, black must be in order of pattern. While the white is obtained from 6 × 6-6 = 30. So it is also for other patterns sir.

P : So, if the pattern is at n?
S : The n pattern is just like this, sir, if the 20th pattern means black 20, white 20 × 20-20 = 380, if the n pattern, I cannot do it sir.



H (Black Colour)	**P** (White Colour)
1st Pattern = 1 } +1	0 = 1
2nd Pattern = 2 } +1	2 = 4
3rd Pattern = 3 } +1	6 = 9
4th Pattern = 4	12 = 16
6th Pattern = 6	30 = 36
10th Pattern = 10	90 = 100

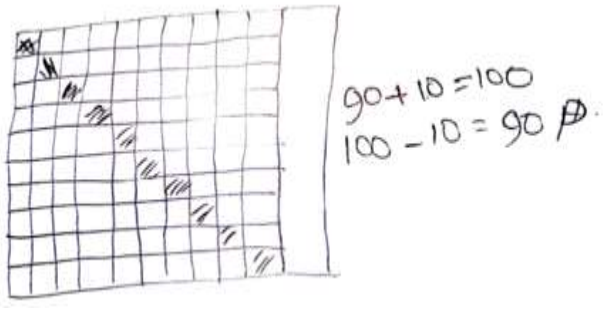


Figure 5. The Worked of S5

The difficulty in formulating the nth pattern is also seen in the S6 subject. The subject understands that the number of black and white paving is n × n, but has difficulty in formulating the nth pattern to find out the respective amounts of black and white paving. The following are excerpts from the interview result of the researcher and the S6 subject and the result of the S6 subject in formulating the n-th pattern.

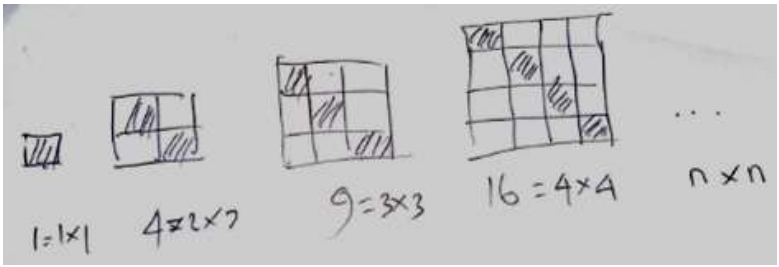


Figure 6. The worked of S_6

5. Conclusion

Based on the results and discussion of the study that the difficulties experienced by students in the process of conjecturing are outlined here: 1) difficulty in interpreting the pattern of n, 2) difficulty in finding the regularity of the pattern, 3) The difficulty in finding and predicting the next pattern based on regularity, 4) Difficulty in Forming Pattern of n. For further research, it is suggested to do research that may help the difficult students in conjecturing process. The natural condition of the students should be overcome by doing research on scaffolding to help students overcome difficulties in the conjecturing process.

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