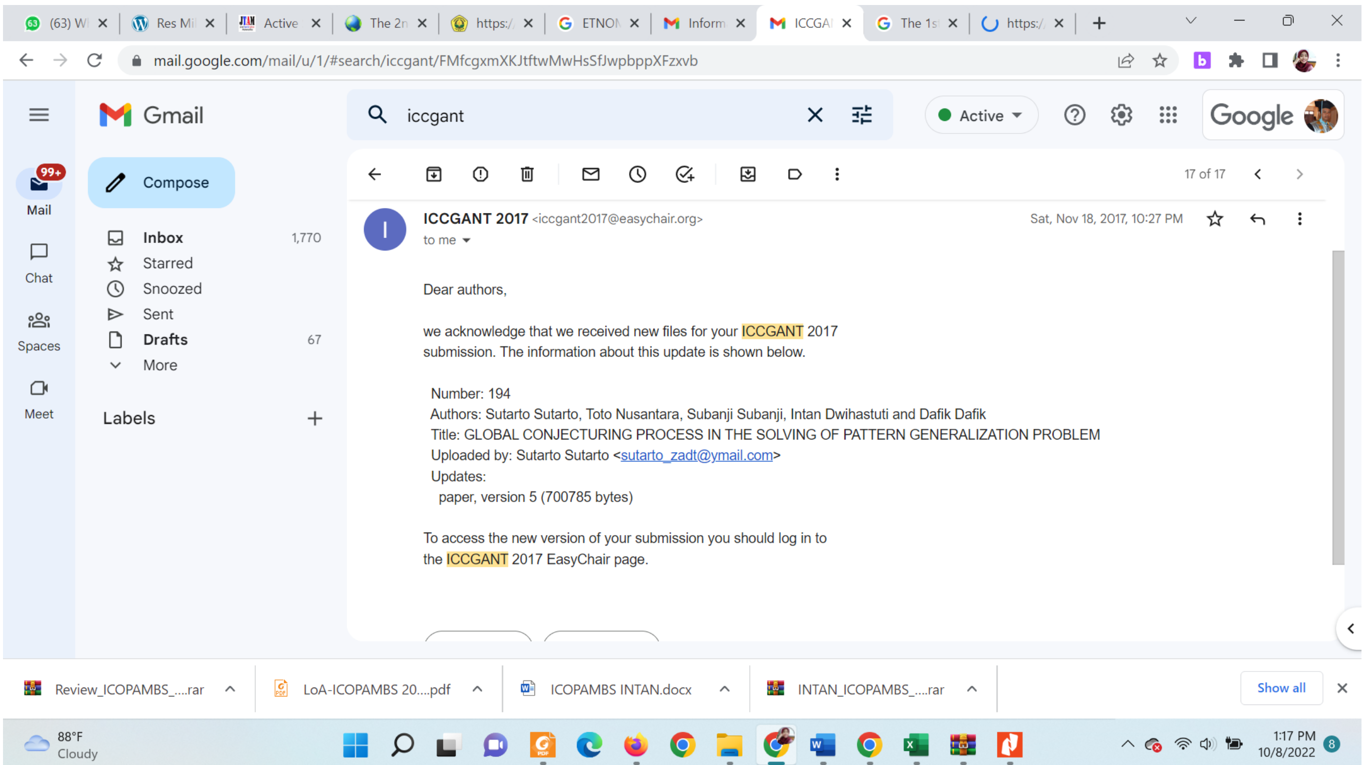
GLOBAL CONJECTURING PROCESS IN PATTERN GENERALIZATION PROBLEM

The 1st International Conference of Combinatorics, Graph Theory, and Network Topology



The paper indicates poor mastery of written English as various mistakes in grammar, cohesion, style, and convention are present throughout the paper. Also, the paper has yet to establish sound issue to delve into, nor has it been designed with consideratem ethodological framework. The discussion and conclusion, thus, result in insignificant implication, as viewed from the objective.

Global Conjecturing Process in ~~the Solving of Pattern~~ Generalization Problem

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Abstract. The aim of this st~~udy to describe t~~he global conjecturing process i~~n the solving of pattern generalization pro~~blem based on APOS theory. The subjects of this study are 15 of 8th grade of Junior High School students. Data collection used Pattern Generalization Problem (PGP) and interviews. In the first stage, students completed PGP; in the second stage, work-based interviews were conducted by the researchers to understand the process of conjecturing. These interviews were video tapped. The global conjecturing process occurs at the stage of action in which subjects build a conjecture by observing and counting the number of squ~~ares complete,~~ at~~ the stage of pro~~cess, the obj~~ect and scheme were perfectly perform~~ed.

1. Introduction

One of the assessment standards since pre – Kindergarten until Senior High School in NCTM (2000) is making and examining the ma~~thematics conjecture~~. Furthermore, it is explained that making conjecture is important because it functions as the basic to develop new perception and increase the further study. Making and examining conjecture is one of the stages of m~~athemat~~ic (Lakatos, 1976), reasoning (Reid, 2002), and mathematical thinking (Mason, Burton, & Stacey, 2010, p. 58).

Conjecture is the logical statement which its truth is not yet certain, (Canada & Castro, 2005; Fischbein, 2002; Mason, Burton, & Stacey, 2010; Reid, 2002). Along with this idea, Cañadas, Deulofeu, Figueiras, Reid, and Yevdokimov (2007) stated that conjecture is a statement concerning all the possible cases, based on the empirical facts, but with the doubtfulness element. Based on these arguments, it can be said that conjecture is the resulting statement the reasoning process which its truth is not yet certain.

Conjecture and problem solving are ~~the~~ linked activity. Cañadas, Deulofeu, Figueiras, Reid, and Yevdokimov (2007) stated that conjecture and problem solving are the important parts and interrelated in the mathematics activity. Moreover, it is said that the problem solving involves the finding, conjecture is the main road for the finding (NCTM, 2000). In problem solving, conjecture helps the problem solver to find the solution from the problems faced. The resulting Conjecture does not just show up, but there is a process and the process is the conjecturing process.

The conjecturing process is the process constructing the conjecture (Cañadas, et al., 2007; Mason, Burton, & Stacey, 2010). Fishbein (2002) assumes that the conjecturing process is the mental activity expression in problem solving based on the knowledge which has been owned and the trust is necessary to be proven. Based on the argument, it can be concluded that the conjecturing process is the mental activity in constructing conjecture based on the possessed knowledge. The mental activity is the process in the mind which can be seen by the students' behavior in problem solving (Hastuti, et al., 2016).

There will be the conjecturing process if the students face any problems. In the conjecturing process, the students construct the conjecture based on the possessed knowledge. The conjecture built by the students, then there will be the validation but also there is also no validation, it depends on the students involved. If there is a validation for the conjecture which is built, so the conjecture is considered to be the correct. If the conjecture is validated so the conjecture can be correct or incorrect. Then, if the conjecture has the incorrect value so it will be done the conjecture constructing process again to result in the new conjecture until the conjecture has the correct value. The conjecture with the correct value is the solution for the problems faced by the students. The conjecturing process illustration is presented in the figure 1 below.

Theoretical framework backgrounding this conjecturing process is missing.

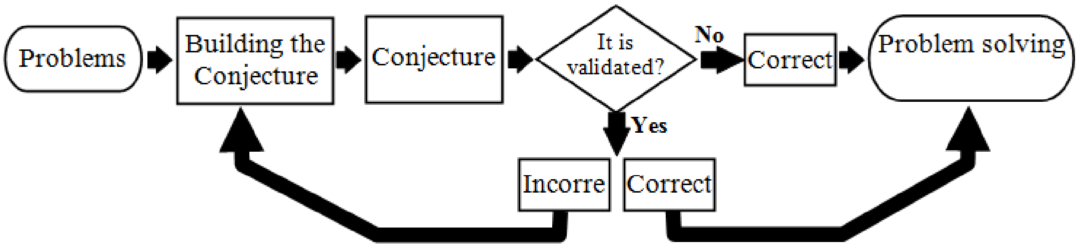


Figure 1. The conjecturing process illustration

Related to the conjecturing process, Cañadas, Deulofeu, Figueiras, Reid, and Yevdokimov (2007) assume that one of the familiar conjecturing processes in mathematic problem solving is the conjecturing type of empirical induction from a finite number of discrete cases. This type of conjecturing consists of seven stages namely observing the case, organizing the case, finding and predicting the pattern, formulating the conjecture, validating the conjecture, generalizing the conjecture, and validating the generalization. The conjecturing process type of empirical induction from a finite number of discrete cases is mostly found in the problem related to the numbers, which the pattern observed is consistent. In the problem solving involving the numbers with consistent pattern, the seven conjecturing processes do not always happen, there are many factors affecting such as the type of task or the students' characteristics involved (Canadas, 2005).

Mulligan and Mitchelmore (2009) mention that the pattern described as the regularity which can be predicted, commonly involves the numerical, spatial or logical relations. Many mathematicians state that the mathematics is a 'science about pattern' (Resnik, 2005; Tikerar, 2009). Resnik and Tikerar highlight the pattern existence in all mathematic fields. Especially, the pattern is considered by some researchers as the way used to reach the algebra because the pattern is the basic measure to construct the generalization which is the mathematic essence (Zazkis & Lijedahl, 2002).

The generalization of pattern is the important aspect in school mathematical activities (Dindyal, 2007; Mulligan, Mitcelmore, English, & Robertson, 2011; Vogel, 2003; Zazkis & Liljedahl, 2002). Along with this ides, Küchemann (2010) stated that the generalization must be the core of the school mathematical activity. The generalization of pattern is the activity making the pattern common rule based on the special examples, the common rule obtained is the conjecture. Yerushalmy (1993) stated that the generalization is the specific type of conjecture, obtained from the special to common reasoning.

The significant contribution to the conjecture or conjecturing has been studied by researchers. ~~Among other~~ is Fischbein (2002) considers the conjecture as the intuition expression. Mason (2002) shows the important of conjecturing atmosphere. Research has been carried out on the production of conjectures within a dynamic geometry environment (Furinghetti & Paola, 2003). Bergqvist (2005) analyzes how the students verify the conjecture and how the teachers' trust related to this process. Lee and Sriraman (2010) develop an open classical analogy in geometry constructing. Lin (2006) designing mathematics conjecturing activities to foster thinking and constructing actively. Then, Cañadas, Deulofeu, Figueiras, Reid, and Yevdokimov (2007) analyze various familiar types and stages of conjecturing process in problem solving. Among the studies, it is not revealed yet on how the students; conjecturing process in generalization of pattern problem solving.

The students' conjecturing process in generalization of pattern problem solving is grouped into two types, namely global conjecturing, and local conjecturing. The global conjecturing is the mental activity in constructing the conjecture by observing the problems intact, and The local conjecturing is the mental activity in constructing the conjecture by observing the problems separately. The global conjecturing process is often done by the students in generalization of pattern problem solving. Thus, this study will describe the global conjecturing process in generalization of pattern problem solving based on APOS theory. The justification of the research objective is not yet sound as it has weak theoretical and empirical supports.

The mental activity in conjecture is analyzed using APOS theory, because APOS theory is a theory which can be used as the analytical tool to describe one's scheme development in a mathematic topic which is the totality of the related knowledge (aware or unaware) to the topic (Dubinsky, 2001). This theory is based on the hypothesis that one's mathematic knowledge will tend to solve the situation as the mathematical problem by constructing the action, process and object as well as regulating the scheme to comprehend the situation and solve the problem (Dubinsky & McDonald, 2001). This theory is called as the APOS theory describing an action at the interiorization as the Process. The Process is encapsulated in an object. Then, it is related to other knowledge in a schema. A schema can also be encapsulated as an object.

2. Theoretical background

Theoretical background is not necessary as it can be included in the first section to establish more significant background to the study.

2.1. Indicators of global conjecturing process

Describing a global conjecturing process in generalization of pattern problem solving is the goal of this study. The global conjecturing process described follows the theories (Canada & Castro, 2005; Cañadas, et al., 2007; Polya, 1967; Reid, 2002). These theories are the basic of conjecturing process of type of empirical induction from a finite number of discrete cases.

Polya (1967) shows four inductive reasoning processes in problem solving, namely (1) observing specific cases, (2) formulating the conjecture based on previous case, (3) generalization, and (4) conjecture verification with specific new cases. Reid (2002) uses the inductive reasoning process in the context of empirical induction from a finite number of discrete cases by the stages: (1) observing specific cases, (2) observing the pattern, (3) formulating the conjecture for common cases (with doubtfulness), (4) generalization, and (5) using generalization to prove. Then Canada and Castro (2005) states that there are seven stages in describing the inductive reasoning process, namely (1) Observing cases, (2) Organizing cases, (3) Searching for and predicting patterns, (4) Formulating a conjecture, (5) Validating the conjecture, (6) Generalizing the conjecture, (7) Justifying the generalization.

Cañadas, Deulofeu, Figueiras, Reid, and Yevdokimov (2007) use seven stages in describing the inductive reasoning process from Canadas as one of the types of conjecturing process namely empirical induction from the finite number of discrete cases. The term of conjecturing process meant in this study is the conjecturing process of type of empirical induction from a finite number of discrete cases. The explanation of the seven conjecturing process stages (Sutarto, et al., 2016). Based on the explanation of the seven stages and indicator of conjecturing processes adapted from Sutarto et al. (2015).

2.2. Generalization of Pattern

Many mathematicians say that mathematics is ‘a science about pattern’ (Resnik, 2005; Tikerar, 2009). Studying about pattern is something important and necessary to be taught since early age. NCTM (2000) recommends that the students participate in pattern activity since the early age, with the hope that they can (1) make the generalization about geometrical and numerical pattern (2) provide a justification for their conjecture, (3) state the pattern rules and the function of verbal, table, and graph forms. Based on some of these opinions, it can be concluded that the pattern is an important matter to be taught from an early age due to it can train the children in learning to reason.

The generalization of pattern is an activity to make a common rule of pattern based on specific examples. The specific examples can be shaped of verbal, symbolic, numeric, and graphical (Bossé, Gyamfi, & Cheetham, 2011). In generalizing patterns, it is not enough to declare a common rule and pattern order verbally but should state the common rule of pattern with symbol. To answer the research questions, then the information provided by specific cases is expressed in a linear pattern in the form of graphics, because the graphical pattern form allows the students to think more complex. Specifically, the pattern is seen by some researchers as the method used to arrive at algebra because the pattern is a fundamental step to construct the generalization which is the essence of mathematics (Zazkis & Lijedahl, 2002).

2.3. Problem of Research

How is the global conjecturing process in the solving of pattern generalization problem based on APOS theory?

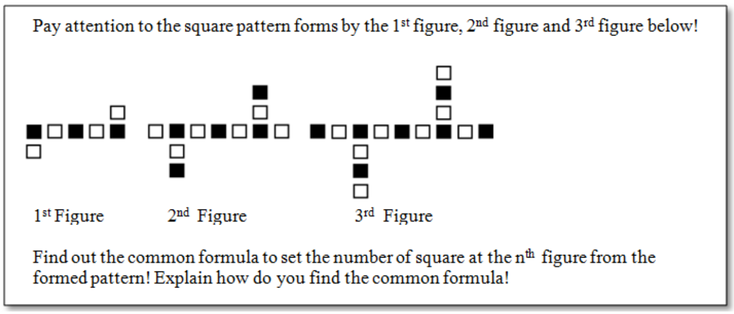
3. Methodology of Research

3.1. Subjects

The subjects in this study are 15 students of class VIII derived from 9 students of VIII State Junior High School 1 Malang, and 6 students of State Junior High School 3 Malang.

3.2. Instrument

There are two types of instruments used, ma~~in and auxiliary instrumen~~ts. The main instrument is the researchers themselves who act as planners, data collectors, data analysts, interpreters, and reporters of research results. The auxiliary instrument used in this study is a Pattern Generalization Problem (PGP) and interviews. The problem given aims to obtain a description of the process of conjecturing of the students, while the interview used was unstructured interview. The PGP is presented in Figure 2.



More details regarding the interview guideline are required

Figure 2. The Pattern Generalization Problem (PGP)

3.3. Data Analysis

This study is a qualitative research with descriptive exploratory approach. At the data analysis stage, the activities conducted by researchers were (1) transcribing the data obtained from interviews, (2) data

The design for data analysis has yet to be clear as there is no detail concerning the data to gain in the study.

The research method is weak due to unclear justification to the research design.

condensation, including explaining, choosing principal matters, focusing on important things, removing the unnecessary ones, and organizing raw data obtained from the field (3) encoding the data from PGP answer sheet and interviews refer based on indicators of local conjecturing process are presented in Table 1, (4) describing the global conjecturing process in the solving of pattern generalization problem based on APOS theory, and (5) conclusion..

4. Results of Research

Based on the analysis results of the answer sheets and the interview results, it is obtained the data on the global conjecturing process conducted by the students in the generalization of pattern problem solving based on the APOS theory. After getting bored for the subject taking process, it is obtained 6 subjects who conduct the global conjecturing process, 5 subjects who conduct the contrast conjecturing and 3 subjects who conduct the local conjecturing and generalizing symbolic. Out of 6 subjects, it will be described two subjects making the global conjecturing process of generalization of pattern problem-solving which is the S1 subject and S2 subject. The data presented is obtained by the procedures (1) the subjects complete the PGP, and (2) after the subjects complete the PGP, then they are interviewed to explore about the global conjecturing process which has been conducted. The data presentation and analysis of the global conjecturing processes in the generalization of pattern problem solving is as the following.

4.1. S1 subject data presentation

In generalizing the S_1 patterns, it has realized that 1st figure, 2nd figure, and 3rd figure form a pattern. To find a common formula on the number of square at the n^{th} figure, S_1 observes and counts the number of square regardless the black square and white ones at the 1st figure, 2nd figure, and 3rd Figure Here are the interview quotation and the S_1 work results in completing the following PGP.

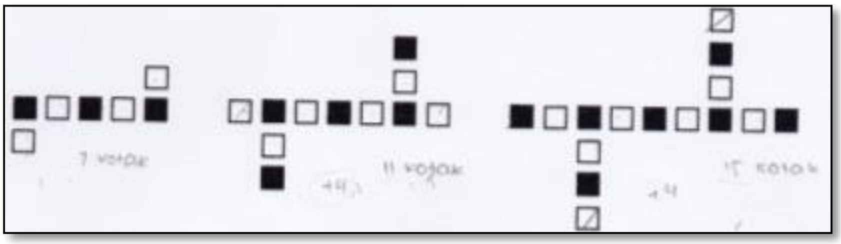


Figure 3. S_1 subject work result

$S_1$04 : this is the different of the figure, Sir. This is the first, 7 and the second one is 11, and the third one, the number is 15 (while pointing to the square figure). The different is 4, so the following figure is plus 4, plus 4, plus 4.

Based on the number of square in the 1st figure, 2nd figure, and 3rd figure, S_1 organizes the cases by ordering the number row pattern. Then, S_1 finds and predicts the pattern by seeing the different between the 2nd figure and the 2nd figure, the 3rd figure and the 2nd figure and thinking how the following figure is plus 4, plus 4, plus 4. This is confirmed by the $S_1$04 interview quotation and the students' work results in completing the following PGP.

Figure 4. S_1 subject work result

To formulate the conjecture, S_1 subject sees the addition of 1st figure into the 2nd figure is 2, and the 2nd figure into the 3rd figure is also 4, by seeing this addition, S_1 formulates the n^{th} formula conjecture in setting the number of square at the figure is $n = n + 4$. After that, S_1 validates the conjecture by seeing the appropriateness with the number of square at the 4th figure and 3rd figure, then saying that the nth formula is incorrect. The following is the S_1 interview quotation.

$S_1$08 : this adds by 4, then adds by 4, then adds by 4. But, thinking it continuously, it may be incorrect. It is why, when 4 adds by 4, it is 8, then when n is 3, it adds by 4 the result is only 7. So it is incorrect.

After realizing that the conjecture formulated is incorrect, S_1 tries new strategy to formulate the conjecture namely by finding the initial number before it is plus 4 because the pattern always adds by 4. S1 finds the initial number by looking for the different of the number of the square at the 1st figure, 2nd figure and 3rd figure by 4. The subtraction results consecutively are 3, 7, 11. S1 realizes that the initial number searched is not yet correct because its initial number is still different. This is shown by the interview quotation and the S_1 work results as the following.

$S_1$11 : this is initially still different (3, 7, 11) meaning that it is incorrect (while pointing out the work result)

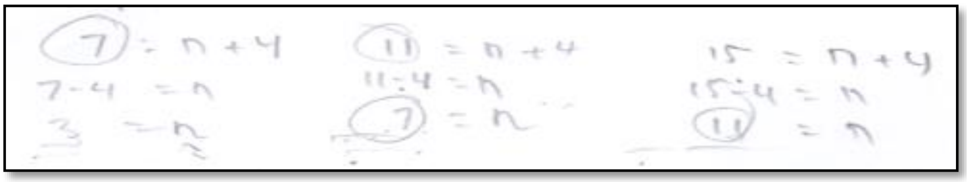


Figure 5. S_1 subject work result

S_1 subject then uses the new strategy which is to look for the initial numbers before adding by 4 times n. S1 writes down the initial number symbols with x, for the 2nd figure $x + (4 \times 2) = 11$, then $x = 3$, for the 3rd figure $x + (4 \times 3) = 15$ then $x = 3$ so the initial number before plus 4 times n is 3. After finding the initial figure, S_1 formulates the conjecture namely the common formula which is $3 + (4 \times n)$and validates the conjecture based on the number of squares which are already known. After validating S_1 generalizes the conjecture as to believe that the common formula is $3 + (4 \times n)$It is also shown from the quotation interview and S_1 work result as the following.

$S_1$11 : I look for the intial number before it is plus 4 times n. the second figure is the same to $x + (4 \times 2) = 11$ so $x = 3$. Then the 3rd figure is similar to $x + (4 \times 3) = 15$ so $x = 3$. So, the intial number before it is plus 4 times n is 3.
P 13 : Okay, then are you sure by the common formula you obtain?
$S_1$13 : yes, Sir I am…

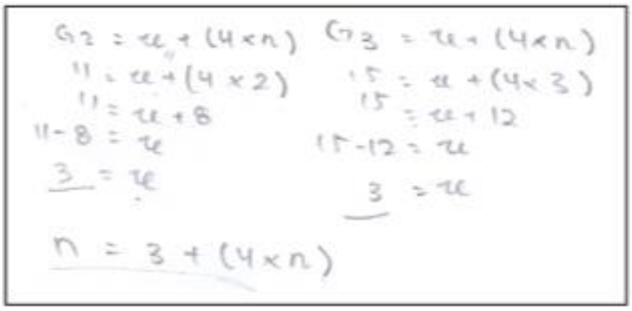


Figure 6. S_1 subject work result

S_1 justifying the generalization with the aim to convince others that the conjecture obtained is correct with a particular example. S_1 describes how to obtain the formula, and shows an example of the formula suitability with a number for a square at the 1st, 2nd, 3rd figures and calculate the number of square at the 4th figure like what has done at the validation stage which $n = 3 + (4 \times 4) = 19$ and 19 is also obtained from the 3rd figure plus 4 is 19, from this example S_1 justifying the resulting generalizations. This is shown by the interview quotation as the following.

P 19 : Okay, then how do you explain that the resulting *formula is correct?*
$S_1$19: I will explain how I get the formula and show the example for the 1st, 2nd, 3rd and 4th figures. For example, for the 4th figure, $n = 3 + (4 \times 4) = 19$. *19 is also obtained from the 3rd fig.,* $15 + 4 = 19$. *(while pointing put the work). s*

From the data described based on the conjecturing process steps, it can be described the S_1 subject thinking structure analyzed based on the APOS stage. The S1 conjecturing process in the generalization of pattern problem solving begins with the action stages namely observing case, and organize the case, then S_1 internalizes the action into prose by finding and predicting the pattern. Once internalized the action into the process, S_1 encapsulates the process into the object by formulating the conjecture and validating the conjecture. At the following scheme stage, S_1 generalizes the conjecture and justifying the resulting conjecture. S_1 thinking structure is presented in Figure 7.

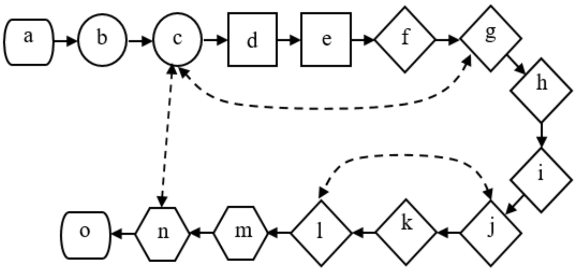


Figure 7. S_1 subject thinking structure

Notes:

a :	The problem proposed is to find the common formula to set the number of square at nth figure	l :	The nth formula is $3 + (4 \times n)$
b :	Counting and observing the number of square at the 1st, 2nd, and 3rd figures.	m :	Believing in that The nth formula is $4n + 3$
c :	Counting the number of square at the 1st, 2nd, and 3rd figures	n :	Validating the nth formula by pointing out at the example at the 1st, 2nd, 3rd, and 4th figures, supposed the 4th figure $n = 3 + (4 \times 4) = 19$. 19 is also obtained from the 3rd figure, $15 + 4 = 19$.
d :	Writing down the row pattern of 7, 11, 15	o :	Done
e :	Counting the square different at the 1st, 2nd, and 3rd figures and thinking of the following object	$\longrightarrow$	Activity sequence
f :	Stating the row different is 4	$\leftarrow - \rightarrow$	Validation activity, for example from g to c, then go back to g; from l to i, then go back to l, etc
g :	The addition is 4 so $n = n + 4$	○	Action
h :	Finding the initial number before being added by 4.	□	Process

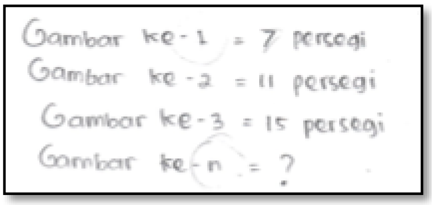
i :	Counting the different of number of square at the 1st, 2nd, 3rd and 4th figures, the results are 3, 7, 9.	◇	Object
j :	$x + (4 \times 2) = 11, x = 3$ $x + (4 \times 3) = 15, x = 3$	⬡	Schema
k :	The initial number before 4 times n is 3	▢	Initial and final activities

4.2. S2 subject data presentation

In generalizing the patterns, S_2 subject has been aware that the 1st figure, 2nd figure and 3rd figure form a pattern. To find a common formula of the number of square at the nth figure, S_2 observes and counts the number of square regardless the black square and white one at 1st figure, 2nd figure and 3rd Figure Here is the interview quotation of S_2.

P 04 : what do you think first when reading this problem?
$S_2$04 : at first I look at the figure, then from this figure, I look at another figure continuously, then it compare both (while pointing out at the PGP)
P 05 : Then you compare, what does it mean?
$S_2$05 : ...When comparing both, I find if in each figure there is 4 addition, four square addition. I still can not see the white and black. I don't see it. Then at first, I think of this continuously, the pattern is always like this (while pointing out at the PGP)

Based on the number of square at the that the 1st figure, 2nd figure and 3rd figure, S_2 subject organizes cases by signing up to number one with the 1st figure, number two with the 2nd figure, number three with the 3rd figure, and so on. Furthermore, S_2 locates and predicts the patterns by comparing the number of squares at the 1st figure, 2nd figure and 3rd figure and finds that the number of additional figure is always 4 and thinks of that the pattern always continues. This is confirmed by $S_2$05 interview quotation and the student's work results in completing the following generalization of pattern problem solving.



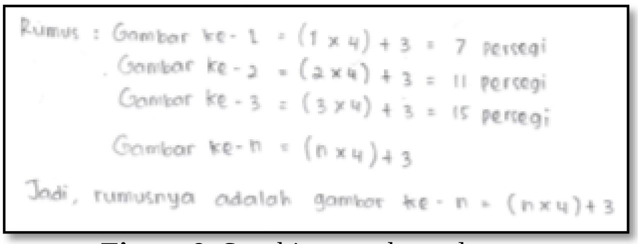
1st figure = 7 squares
2nd figure = 11 squares
3rd figure = 15 squares
Nth figure = ?

Figure 8. S_2 subject work result

To formulate a conjecture S_2 tries to determine suitable n based on the figure sequence. For example, 7 squares and 11 squares, this means that it has to plus 4, if n then n + 4 it can not be. So it must determine a suitable n based on the figure sequence. After S_2 tries to enter 1st figure (one) into the formula because one is also n, by trying one by one starting from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$. Then, S_2 formulates a common formula of conjecture to determine the number of square at the nth figure $= (n \times 4) + 3$ and validates the conjecture based on the number of squares at the 4th figure and 5th Figure After validating, S1 generalizes the conjecture as to believe that the common formula is $= (n \times 4) + 3$. It is also shown from the interview quotation and the students' work in completing the following generalization of pattern problem solving.

$S_2$08 : so, I try the first one, supposed 1×1, 1×1 must be added with what number to be 7, eee.. then it is plus 6, but if it is supposed $2 \times 1 + 6$ then the results is not 11, so I keep trying the closest ine which is the most appropriate answer for this square ((while pointing out at the square figure at PGP). I keep trying $(1 \times 2) + 5 = 7$ is correct, then this one $(2 \times 2) + 5$ the result is 9 and not 11, so I keep trying four $(1 \times 4) + 3 =$ this 7, I try if it is $(2 \times 4) + 3 = 11$, I try again the

$(3 \times 4) + 3 = 15$, *because Iam still doubt I try this one* $(4 \times 4) + 3 = 19$, $15 + 4 = 19$. *So that is my thinking pattern.*


Rumus : Gambar ke-1 = (1 x 4) + 3 = 7 persegi
Gambar ke-2 = (2 x 4) + 3 = 11 persegi
Gambar ke-3 = (3 x 4) + 3 = 15 persegi
Gambar ke-n = (n x 4) + 3
Jadi, rumusnya adalah gambar ke-n = (n x 4) + 3

Formula : 1^{st} figure = (1 x 4) + 3 = 7 squares
2^{nd} figure = (2 x 4) + 3 = 11 squares
3^{rd} figure = (3 x 4) + 3 = 15 squares
n^{th} figure = (n x 4) + 3
So, the formula is the n^{th} figure is (n x 4) + 3

Figure 9. S_2 subject work result

S_2 subject justifying the generalization with the aim of convincing others that the resulting conjecture is correct with a particular example. S_2 points out, from this example, S_2 justifying the generalization results in. This is shown by the following interview quotation.

P 14 : Okay, then how do you explain to others that the resulting formula is correct?
$S_2$14: I will show the results at the 1^{st}, 2^{nd}, 3^{rd} figures and so on. This is the proof (while pointing out the work result). I have tried it continuously, then it is correct. So I believe in the formula.

From the data described based on the conjecturing process steps, it describes S_2 subject thinking structure which is analyzed based on the APOS stage. S_2 Conjecturing process in the generalization of pattern problem solving begins by observing case the action stage, and organizing the case, then S_2 internalizes the action into prose by finding and predicting the pattern. Once internalized into the action, S_2 encapsulates the action into the by formulating the conjecture and validating the conjecture. Then at the following schema stage, S_2 Generalizing the conjecture and justifying the resulting conjecture. S_2 thinking structure is presented in Figure 10.

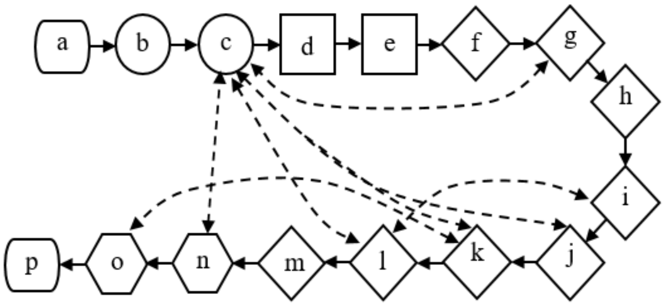


Figure 10. S2 subject thinking structure

Notes:

a :	The problem proposed is to find the common formula to set the number of square at n^{th} figure	m :	The n^{th} formula is $(n \times 4) + 3$
b :	Counting and observing the number of square at the 1^{st}, 2^{nd}, and 3^{rd} figures.	n :	Believing in that The n^{th} formula is $(n \times 4) + 3$
c :	Counting the number of square at the 1^{st}, 2^{nd}, and 3^{rd} figures	o :	Validating the n^{th} formula by specific case
d :	Making list of table to sort the pattern of 7, 11, 15	p :	Done
e :	Counting the square different at the 1^{st}, 2^{nd}, and 3^{rd} figures and thinking of the following object	$\longrightarrow$	Activity sequence

f :	Stating the row different is 4	◄- - ►	Validation activity, for example from g to c, then go back to g; from l to i, then go back to l, etc
g :	Looking for suitable n to sort the 1st, 2nd, and 3rd figures	○	Action
h :	Supposed, $7 + 4 = 11$, if $n + 4$ it can not be	□	Process
i :	Trying to input 1st (one) figure into the formula because one is n	◇	Object
j :	Trying (1×1) adds with what number to be 7. $(1 \times 1) + 6 = 7$ correct, $(2 \times 1) + 6 = 8$ incorrect	⬡	Schema
k :	Trying (1×2) adds with what number to be 7. $(1 \times 2) + 5 = 7$ correct, $(2 \times 2) + 5 = 9$ incorrect	▢	Initial and final activities
l :	Trying (1×4) adds with what number to be 7. $(1 \times 4) + 3 = 7$ correct, $(2 \times 4) + 3 = 11$ correct $(3 \times 4) + 3 = 15$ correct, $(4 \times 4) + 3 = 19$ correct		

4.3. Global Conjecturing Process Schema of S1 subject and S2 subject in Generalization of Pattern Problem Solving based on APOS

In generalizing the patterns, S_1 and S_2 have been aware that the 1st figure, 2nd figure and 3rd figure form a pattern. To find a common formula of the number of square at the nth Figure At this action stage, S_1 and S_2 observe and count the number of square regardless the black square and white one, at the 1st figure, 2nd figure and 3rd Figure Based on the number of square at the 1st figure, 2nd figure and 3rd figure, S_1 organizes cases by sorting the number row patterns and S_2 registers to relate number one with the 1st figure, number 2 with 2nd figure, number 3 with the 3rd figure, and so on. Then, at the process stage, S_1 and S_2 are searching for and predicting the pattern by looking at the difference between the 2nd figure and the 1st figure, 3rd figure and the 2nd figure and think that the following figure increases by 4.

The *object* stage, to formulate conjecture, S_1 sees the 1st figure addition to the 2nd figure is 4, and the 2nd figure to the 3rd figure is also 4, by looking at the addition, S_1 looks for the initial number before adding by 4 times n. For the 2nd figure $x + (4 \times 2) = 11$, then $x = 3$, for the 3rd figure $x + (4 \times 3) = 15$ then $x = 3$ so the initial number before added 4 times n is 3. S_2 Subject tries to determine the suitable n based on the figure sequence, then S_2 tries to enter the 1st (one) figure into the formula because one is n, by trying one by one starting from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$. The conjecture produced by S_1 and S_2 is $3 + (4 \times n)$ and validate the conjecture based on specific examples obtained at the action or process stage.

The scheme stage, S_1 and S_2 subjects generalize the conjecture to believe that the conjecture resulted is correct after validating the conjecture in the previous stages. In justifying the generalization with the aim of convincing others that the resulting conjecture is correct, S_1 and S_2 use specific examples obtained at the action stage or process stage. Justifying the generalizations made by the subject S_1 and S_2 is the same as what has done at the validation stage namely using the specific examples. The Global conjecturing process scheme of S_1 subject and S_2 subject in the solving of pattern generalization problem is presented in Figure 11.

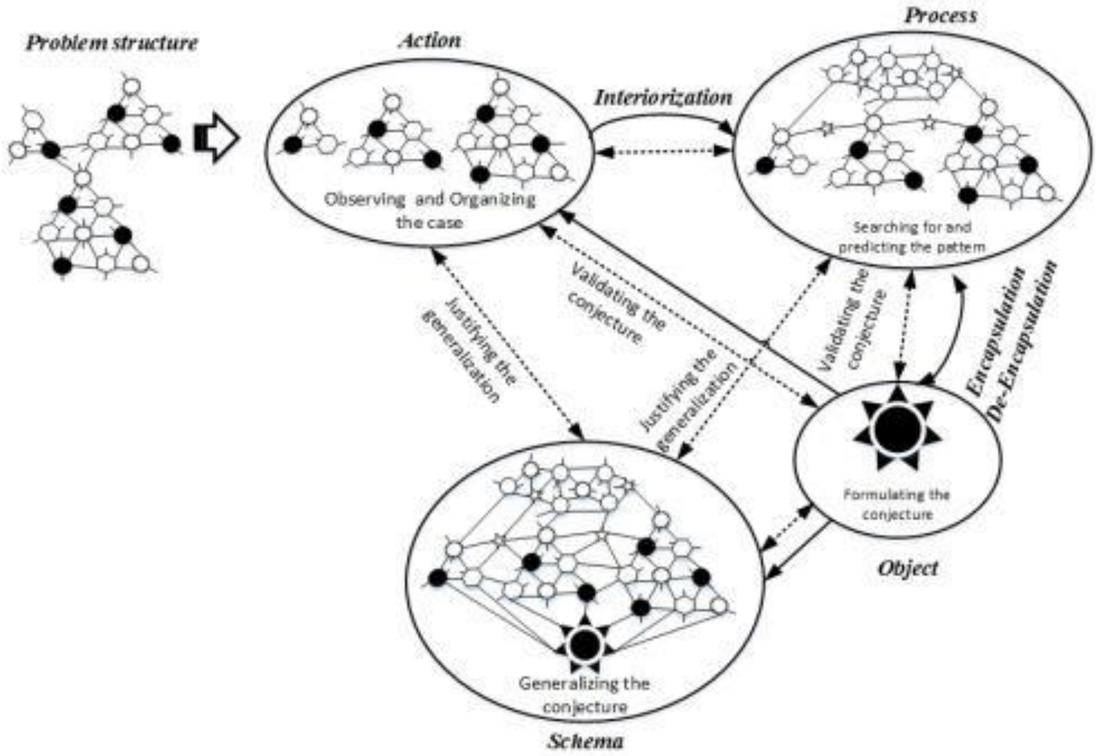


Figure 9. The schema of global conjecturing process

5. Discussion

In this section we will discuss the research findings related to global conjecturing process in the generalization of pattern problem solving. In generalizing the pattern, S_1 and S_2 on the "action" stage have realized that the 1st figure, 2nd figure, and 3rd figure form a pattern. To find a common formula of to the number of square in the nth , S_1 and S_2 observe by counting the number of square regardless the black and white squares on the 1st figure, 2nd figure, and 3rd figures. Based on the number of square at the 1st figure, 2nd figure, and 3rd figure, S_1 organizes the case by sorting the row pattern of 7, 11, 15 and so on while S_2 makes a list or a table to relate number one with the 1st figures, number 2 with the 2nd figure, number 3 with the 3rd figure, and so on. This shows that at the "action" stage, S_1 subject and S_2 subject observe the cases and organizes the cases regardless the black and white squares, therefore the conjecturing process conducted by the subjects is referred to as the global conjecturing process. Observing cases and organizing cases regardless the black and white squares are based on one of the Gestlat laws in the observation namely Law of similarity which is a law which person tends to perceive the same stimulus as a whole (King & Wertheimer, 2009).

At this "process" stage, the subjects internalize the action to find and predict the pattern by looking at the difference, or the difference between the number of square at the 2nd figure and 1st figure, 3rd figure and the 2nd figure and think that the following figure has the same pattern, namely obtaining the increased 4. In formulating the conjecture stage, S_1 conducts the encapsulation to generate the object which is to see the 1st figure addition to the 2nd figure is 4, and the 2nd figure to 3rd figure is also 4. S_1 seeks the initial number before adding 4 times n. for the 2nd figure, -2 $x + (4 \times 2) = 11$ so $x = 3$, for the 3rd figure -3 $x + (4 \times 3) = 15$ so $x = 3$ so the initial number before being added 4 times n is 3. After finding the initial number, S_1 formulate the common formula of the conjecture namely $3 + (4 \times n)$ and validating the conjecture based on the number of square known. S_2 tries to det the appropriate n based on the figure sequence, after that S_2 tries to enter the 1st figure into the formula because one is also n, by trying one by trying one by one starting from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$ then S_2

formulates the common formula of the conjecture to set the number of square at the n[th] figure $n = (n \times 4) + 3$ and validate the conjecture based on the number of squares on the 4th figure and 5th figure. The way done by S_1 is looking for the initial number using the x symbol and S_2 seeks the appropriate based on the figure sequence. Both ways are different but meaningful for itself to find the common formula of the conjecture, it describes the knowledge possessed. This is consistent to what expressed by Steinbrig and Yerushalmy (2008) that the mathematical symbol is a tool for coding and describing the knowledge as well as communicating the mathematical knowledge. At this process stage and object stage, the subjects conduct it perfectly.

At this scheme stage, S_1 subject and S_2 subject generalize the conjecture to believe that the resulting conjecture is correct. In justifying the generalization with the aim of convincing others that the resulting conjecture is correct, S_1 and S_2 use the specific examples obtained at the action stage and object stage. S_1 justifying the generalization with the aim of convincing others that the resulting conjecture is correct with the specific example. S_2 counts the number of square at the 4[th] figure namely $n = 3 + (4 \times 4) = 19$ and 19 is also obtained from 3[rd] figure plus 4 is 19, from this example, S_1 validates the resulting generalization. S_2 validates the generalization with aim of convincing others that the resulting conjecture is correct with specific example obtained at the object stage by pointing out $(1 \times 4) + 3 = 7$, $(2 \times 4) + 3 = 11$, $(3 \times 4) + 3 = 15$, and $(4 \times 4) + 3 = 19$. In justifying the generalizations, S_1 subject and S_2 subject conduct their own way, this is based on the Caraher and Martinez (2008) that the students do not just simply use the notation or symbols but also their presentation and give a reason mathematically, make conclusions and generalizations in their way. At this scheme stage, it is also conducted perfectly.

6. Conclusions

Based on the findings in the global conjecturing process conducted by the students in the generalization of pattern problem solving, it increases the conjecturing process theories (Cañadas, Deulofeu, Figueiras, Reid, & Yevdokimov, 2007) about the type of empirical induction from a finite number of discrete cases which has seven stages and not study the students' thinking process in constructing the conjecture generalization. The results show that the global conjecturing process occurs at the stage of action in which subjects build a conjecture by observing and counting the number of squares complete, at the stage of process, the object and scheme were perfectly performed.

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