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By Intan Hastuti

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Global conjecturing process in pattern generalization problem

Sutarto¹, Toto Nusantara², Subanji², Intan Dwi Hastuti³, Dafik⁴

¹Mathematics Education Departement, IKIP Mataram, Mataram, Indonesia

²Department of Mathematics, UM Malang, Malang, Indonesia

³Mathematics Education Departement, Universitas Muhammadiyah Mataram, Mataram, Indonesia

⁴Mathematics Education Departement, University of Jember, Jember, Indonesia

Email: sutarto@ikipmataram.ac.id

Abstract. The aim of this global conjecturing process based on the theory of APOS. The subjects used in study were 15 of 8th grade students of Junior High School. The data were collected using Pattern Generalization Problem (PGP) and interviews. After students had already completed PGP; moreover, they were interviewed using students work-based to understand the conjecturing process. These interviews were video taped. The result of study reveals that the global conjecturing process occurs at the phase of action in which subjects build a conjecture by observing and counting the number of squares completely without distinguishing between black or white squares, finally at the phase of process, the object and scheme were perfectly performed.

1. Introduction

One of the standard assessments used in this study begins from preschool until secondary school level [19] is making and examining the mathematic conjecture. Furthermore, it is explained that making conjecture is important work because it functions as the basic to develop and increase new perception for further study. Making and examining conjecture is a step in mathematical study [12], reasoning [21], and mathematical thinking [16].

Conjecture is the logical statement where its truth is not yet certain [3, 8, 16, 21]. Along with this idea, conjecture is a statement concerning all the possible cases, based on the empirical facts, but with the doubtful elements [4]. Based on these arguments, it can be argued that conjecture is the statement based on the reasoning process where its truth is not yet certain.

Conjecture and problem solving are linked activity. Conjecture and problem solving are the important parts and interrelated in the mathematic activity [4, 2]. Moreover, it is said that the problem solving involves the finding while conjecture is the main road for the finding [19]. In problem solving, conjecture helps the problem solver to find the solution for the problems faced. As a result, conjecture does not just show up, but there is a process and the process is the conjecturing process.

The conjecturing process is the process of constructing the conjecture [18-16]. Assumes that the conjecturing process is the mental activity expressed in problem solving based on the knowledge which has been owned and the trust is necessary to be proven [8]. Based on the argument, it can be concluded that the conjecturing process is the mental activity in constructing conjecture based on the possessed knowledge. The mental activity is the process in the mind which can be seen in the students' behavior in problem solving [11, 30, 31].

There will be the conjecturing process if the students face any problems. In the conjecturing process, the students construct the conjecture based on their knowledge. There is no validation in the conjecture built by the students. It depends on the students' involvement. If there is a validation for the conjecture they built, the conjecture is considered as the correct one. Only after the conjecture is



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validated then one can consider it as correct or incorrect. If the conjecture does not have incorrect value the process of conjecturing will be processed again until the it has the correct value. The conjecture with the correct value is the solution for the problems faced by the students.

In relation to conjecturing process, the familiar one used is mathematic problem solving as it is the conjecturing type of empirical induction from a finite number of discrete cases [4]. This type of conjecturing process consists of seven steps namely observing the case, organizing the case, finding and predicting the pattern, formulating the conjecture, validating the conjecture, generalizing the conjecture, and validating the generalization. The conjecturing process of this type—by inducting from a finite number of discrete cases—is mostly found in the problem related to the numbers, where the pattern under observation is consistent. In the problem solving involving the numbers with consistent pattern, the seven conjecturing processes do not always take place; there are many factors affecting process such as the type of task or the students' characteristics involved [3].

The pattern described as the regularity which can be predicted above commonly involves the numerical, spatial or logical relations [17]. Many mathematicians state that the mathematics is ‘science about pattern’ [22, 25]. They highlight the pattern existence in all mathematic fields [25]. In particular the pattern is considered by some researchers as the strategy applied in algebraic field because the pattern is the basic measure to construct the generalization which is the mathematic essence [29].

Generalization of pattern is an important aspect in school mathematical activities [5, 18, 26, 29]. Along with this idea, the generalization must be the core of the school mathematical activities [10]. The generalization of pattern itself is the activity making the pattern common rule based on some special examples. The common rule obtained is the conjecture, and the generalization is the specific type of conjecture obtained from common reasoning [28].

The significant contribution to the conjecture or conjecturing has been studied by researchers who consider the conjecture as the intuition of expression [8]. It shows the important of conjecturing atmosphere [15]. Research has been carried out on the production of conjectures within a dynamic geometry environment [9] by analyzing how the students verify the conjecture and how the teachers' trust related to this process [1]. Researchers developed an open classical analogy in geometry construction [13] by designing mathematical conjecturing activities to foster thinking and constructing actively [14]. Then, they analyze various familiar types and steps of conjecturing process in problem solving [4]. Among the studies, it is not revealed yet how the students—through conjecturing process—generalizing the pattern of problem solving.

The conjecturing process described above correlates to some theories [3, 4, 20, 21, 23]. These theories are the basic of conjecturing process of empirical induction from a finite number of discrete cases. They consist of four inductive reasoning processes in problem solving namely (1) observing specific cases, (2) formulating the conjecture based on previous case, (3) generalization, and (4) conjecture verification with specific new cases [20]. Reid uses the inductive reasoning process in the context of empirical induction from a finite number of discrete cases by the steps: (1) observing specific cases, (2) observing the pattern, (3) formulating the conjecture for common cases (with doubtfulness), (4) generalization, and (5) using generalization to prove [21]. There are seven steps in describing the inductive reasoning process, namely (1) Observing cases, (2) Organizing cases, (3) Searching for and predicting patterns, (4) Formulating a conjecture, (5) Validating the conjecture, (6) Generalizing the conjecture, (7) Justifying the generalization [3].

There are seven steps in describing the inductive reasoning process drive from Canada's, as such a types of conjecturing process namely empirical induction from finite number of discrete cases. The conjecturing process in this study is empirical induction from a finite number of discrete cases. The explanation of those seven steps take place in conjecturing process [24]. Based on its explanation and indicator adapted from the students, the conjecturing process in generalization of pattern problem solving is grouped into two types, i.e. global conjecturing, and local conjecturing. The global conjecturing is the mental activity in constructing the conjecture by observing the problems intact, and The local conjecturing is the mental activity of constructing the conjecture by observing the problems

separately. While, The global conjecturing process is often done by the students in generalization of pattern problem solving. Thus, this study will describe the global conjecturing process based on APOS theory.

The mental activity in conjecture analyzes using APOS theory, because APOS theory is a theory that can be used as the analytical tool to describe one's developmental scheme in a mathematic topic--as it is the totality of the related knowledge (aware or unaware) to the topic [6]. This theory is based on the hypothesis of mathematic knowledge that may use to solve the situation as the mathematical problem by constructing the action, process and object as well as regulating the scheme to comprehend the situation and solve the problem [7]. This theory is called APOS and use to describe an action at the interiorization as the Process. The Process is encapsulated in an object. Then, it is related to other knowledge a schema. A schema can also be encapsulated as an object. Problem of Research: How is the global conjecturing process in the solving of pattern generalization problem based on APOS theory?

2. Methodology of Research

2.1. Subjects

The subjects in this study 17 students of class VIII derived from 9 students of VIII State Junior High School 1 Malang, and 6 students of State Junior High School 3 Malang.

2.2. Instrument

There are two types of instruments use. The main instrument is the researchers themselves who act as planners, data collectors, data analysts, interpreters, and reporters of research results. The auxiliary instrument used in this study is a Pattern Generalization Problem (PGP) and interviews. The problem given aims to obtain a description of the process of conjecturing of the students, while the interview used was unstructured interview. The PGP is presented in Figure 1.

Pay attention to the square pattern forms by the 1st figure, 2nd figure and 3rd figure below!

1st Figure 2nd Figure 3rd Figure

Find out the common formula to set the number of square at the nth figure from the formed pattern! Explain how do you find the common formula!

Figure 1. The Pattern Generalization Problem (PGP)

2.3. Data Analysis

This study is a qualitative research with descriptive exploratory approach. At the data analysis step, the activities conducted by researchers were (1) transcribing the data obtained from interviews, (2) data condensation, including explaining, choosing principal matters, focusing on important things, removing the unnecessary ones, and organizing raw data obtained from the field (3) encoding the data from PGP answer sheet and interviews refer based on indicators of local conjecturing process are

presented in Table 1, (4) describing the global conjecturing process in the solving of pattern generalization problem based on APOS theory, and (5) conclusion.

3. Results of Research

Based on the analysis results of the answer sheets and the interview results, it is obtained the data on the global conjecturing process conducted by the students in the generalization of pattern problem solving based on the APOS theory. After getting bored for the subject taking process, it is obtained 6 subjects who conduct the global conjecturing process, 5 subjects who conduct the contrast conjecturing and 3 subjects who conduct the local conjecturing and generalizing symbolic. Out of 6 subjects, it will be described two subjects making the global conjecturing process of generalization of pattern problem-solving which is the S₁ subject and S₂ subject. The data presented is obtained by the procedures (1) the subjects complete the PGP, and (2) after the subjects complete the PGP, then they are interviewed to explore about the global conjecturing process which has been conducted. The data presentation and analysis of the global conjecturing processes in the generalization of pattern problem solving is as the following.

3.1. S₁ subject data presentation

In generalizing the S₁ patterns, it has realized that 1st figure, 2nd figure, and 3rd figure form a pattern. To find a common formula on the number of square at the nth figure, S₁ observes and counts the number of square regardless the black square and white ones at the 1st figure, 2nd figure, and 3rd Figure Here are the interview quotation and the S₁ work results in completing the following PGP.

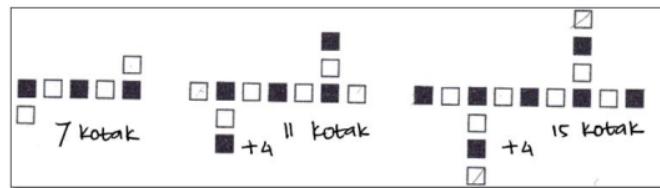


Figure 2.S₁ subject work result

S₁04 : this is the different of the figure, Sir. This is the first, 7 and the second one is 11, and the third one, the number is 15 (while pointing to the square figure). The different is 4, so the following figure is plus 4, plus 4, plus 4.

Based on the number of square in the 1st figure, 2nd figure, and 3rd figure, S₁ organizes the cases by ordering the number row pattern. Then, S₁ finds and predicts the pattern by seeing the different between the 2nd figure and the 1st figure, the 3rd figure and the 2nd figure and thinking how the following figure is plus 4, plus 4, plus 4. This is confirmed by the S₁04 interview quotation and the students' work results in completing the following PGP.

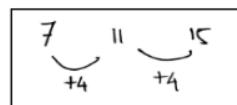


Figure 3.S₁ subject work result

To formulate the conjecture, S₁ subject sees the addition of 1st figure into the 2nd figure is 2, and the 2nd figure into the 3rd figure is also 4, by seeing this addition, S₁ formulates the nth formula conjecture in setting the number of square at the figure is $n = n + 4$. After that, S₁ validates the conjecture by seeing the appropriateness with the number of square at the 4th figure and 3rd figure, then saying that the nth formula is incorrect. The following is the S₁ interview quotation.

S₁08 : this adds by 4, then adds by 4, then adds by 4. But, thinking it continuously, it may be incorrect. It is why, when 4 adds by 4, it is 8, then when n is 3, it adds by 4 the result is only 7. So it is incorrect.

After realizing that the conjecture formulated is incorrect, S₁ tries new strategy to formulate the conjecture namely by finding the initial number before it is plus 4 because the pattern always adds by 4. S₁ finds the initial number by looking for the different of the number of the square at the 1st figure, 2nd figure and 3rd figure by 4. The subtraction results consecutively are 3, 7, 11. S₁ realizes that the initial number searched is not yet correct because its initial number is still different. This is shown by the interview quotation and the S₁ work results as the following.

S₁11 : this is initially still different (3, 7, 11) meaning that it is incorrect (while pointing out the work result)

$\textcircled{7} = n+4$	$\textcircled{11} = n+4$	$15 = n+4$
$7-4 = n$	$11-4 = n$	$15-4 = n$
$3 = n$	$\textcircled{7} = n$	$\textcircled{11} = n$

Figure 4. S₁ subject work result

S₁ subject then uses the new strategy which is to look for the initial numbers before adding by 4 times n. S₁ writes down the initial number symbols with x, for the 2nd figure $x + (4 \times 2) = 11$, then $x = 3$, for the 3rd figure $x + (4 \times 3) = 15$ then $x = 3$ so the initial number before plus 4 times n is 3. After finding the initial figure, S₁ formulates the conjecture namely the common formula which is $3 + (4 \times n)$ and validates the conjecture based on the number of squares which are already known. After validating S₁ generalizes the conjecture as to believe that the common formula is $3 + (4 \times n)$. It is also shown from the quotation interview and S₁ work result as the following.

S₁11 : I look for the initial number before it is plus 4 times n. the second figure is the same to $x + (4 \times 2) = 11$ so $x = 3$. Then the 3rd figure is similar to $x + (4 \times 3) = 15$ so $x = 3$. So, the initial number before it is plus 4 times n is 3.

P 13 : Okay, then are you sure by the common formula you obtain?

S₁13 : yes, Sir I am...

$\begin{aligned} G_2 &= x + (4 \times n) \\ 11 &= x + (4 \times 2) \\ 11 &= x + 8 \\ 11 - 8 &= x \\ 3 &= x \\ \hline n &= 3 + (4 \times n) \end{aligned}$	$\begin{aligned} G_3 &= x + (4 \times n) \\ 15 &= x + (4 \times 3) \\ 15 &= x + 12 \\ 15 - 12 &= x \\ 3 &= x \end{aligned}$
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Figure 5. S₁ subject work result

S₁ justifying the generalization with the aim to convince others that the conjecture obtained is correct with a particular example. S₁ describes how to obtain the formula, and shows an example of the formula suitability with a number for a square at the 1st, 2nd, 3rd figures and calculate the number of square at the 4th figure like what has done at the validation step which $n = 3 + (4 \times 4) = 19$ and 19 is

also obtained from the 3rd figure plus 4 is 19, from this example S₁ justifying the resulting generalizations. This is shown by the interview quotation as the following.

P 19 : Okay, then how do you explain that the resulting formula is correct?

S₁19: I will explain how I get the formula and show the example for the 1st, 2nd, 3rd and 4th figures. For example, for the 4th figure, n = 3 + (4 × 4) = 19. 19 is also obtained from the 3rd fig., 15 + 4 = 19. (while pointing put the work). s

From the data describe ²⁸ based on the conjecturing process steps, it can be described the S₁ subject thinking structure analyzed based on the APOS step. The S₁ conjecturing process in the generalization of pattern problem solving begins with the action steps namely observing case, and organize the case, then S₁ internalizes the action into prose by finding and predicting the pattern. Once internalized the action into the process, S₁ encapsulates the process into the object by formulating the conjecture and validating the conjecture. At the following scheme step, S₁ generalizes the conjecture and justifying the resulting conjecture. S₁ thinking structure is presented in Figure 6.

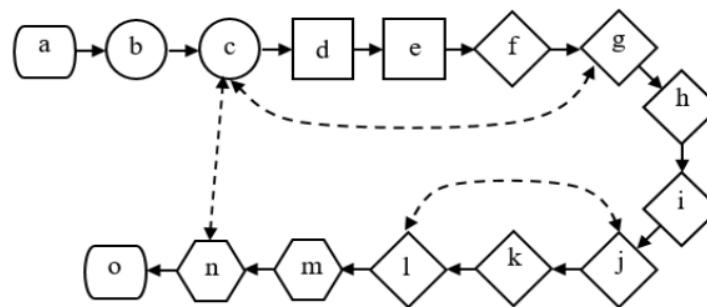


Figure 6. S₁ subject thinking structure

Notes:

a : The problem proposed is to find the common formula to set the number of square at n th figure	1 : The n th formula is 3 + (4 × n)
b : Counting and observing the number of square at the 1 st , 2 nd , and 3 rd figures.	m : Believing in that The n th formula is 4n + 3
c : Counting the number of square at the 1 st , 2 nd , and 3 rd figures	n : Validating the n th formula by pointing out at the example at the 1 st , 2 nd , 3 rd , and 4 th figures, supposed the 4 th figure n = 3 + (4 × 4) = 19. 19 is also obtained from the 3 rd figure, 15 + 4 = 19.
d : Writing down the row pattern of 7, 11, 15	o : Done
e : Counting the square different at the 1 st , 2 nd , and 3 rd figures and thinking of the following object	→ Activity sequence
f : Stating the row different is 4	↔ Validation activity, for example from g to c, then go back to g; from l to i, then go back to l, etc
g : The addition is 4 so n = n + 4	○ Action
h : Finding the initial number before being	□ Process

added by 4.	
i : Counting the different of number of square at the 1 st , 2 nd , 3 rd and 4 th figures, the results are 3, 7, 9.	<input checked="" type="checkbox"/> Object
j : $x + (4 \times 2) = 11, x = 3$ $x + (4 \times 3) = 15, x = 3$	<input checked="" type="checkbox"/> Schema
k : The initial number before 4 times n is 3	<input checked="" type="checkbox"/> Initial and final activities

3.2 S₂ subject data presentation

In generalizing the patterns, S₂ subject 19 been aware that the 1st figure, 2nd figure and 3rd figure form a pattern. To find a common formula of the number of square at the nth figure, S₂ observes and counts the number of square regardless the black square and white one at 1st figure, 2nd figure and 3rdFigure Here is the interview quotation of S₂.

P 04 : what do you think first when reading this problem?

S₂04 : at first I look at the figure, then from this figure, I look at another figure continuously, then it compare both (while pointing out at the PGP)

P 05 : Then you compare, what does it mean?

S₂05 : ...When comparing both, I find if in each figure there is 4 addition, four square addition. I still can not see the white and black. I don't see it. Then at first, I think of this continuously, the pattern is always like this(while pointing out at the PGP)

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Based on the number of square at the that the 1st figure, 2nd figure and 3rd figure, S₂ subject organizes cases by signing up to number one with the 1st figure, number two with the 2nd figure, number three with the 3rd figure, and so on. Furthermore, S₂ locates and predicts the patterns by comparing the number of squares at the 1st figure, 2nd figure and 3rd figure and finds that the number of additional figure is always 4 and thinks of that the pattern always continues. This is confirmed by S₂05 interview quotation and the student's work results in completing the following generalization of pattern problem solving.

Gambar ke-1 = 7 persegi Gambar ke-2 = 11 persegi Gambar ke-3 = 15 persegi Gambar ke-n? ?	1 st figure = 7 squares 2 nd figure = 11 squares 3 rd figure = 15 squares N th figure = ?
---	--

Figure 7. S₂ subject work result

To formulate a conjecture S₂ tries to determine suitable n based on the figure sequence. For example, 7 squares and 11 squares, this means that it has to plus 4, if n then n + 4 it can not be. So it must determine a suitable n based on the figure sequence. After S₂ tries to enter 1st figure (one) into the formula because one is also n, by trying one by one starting from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$. Then, S₂ formulates a common formula of conjecture to determine the number of square at the nth figure = $(n \times 4) + 3$ and validates the conjecture based on the number of squares at the 4th figure and 5thFigure After validating, S₁ generalizes the conjecture as to believe that the common formula is = $(n \times 4) + 3$. It is also shown from the interview quotation and the students' work in completing the following generalization of pattern problem solving.

S₂08 : so, I try the first one, supposed 1×1 , 1×1 must be added with what number to be 7, eee.. then it is plus 6, but if it is supposed $2 \times 1 + 6$ then the results is not 11, so I keep trying the

closest one which is the most appropriate answer for this square ((while pointing out at the square figure at PGP). I keep trying $(1 \times 2) + 5 = 7$ is correct, then this one $(2 \times 2) + 5$ the result is 9 and not 11, so I keep trying four $(1 \times 4) + 3 =$ this 7, I try if it is $(2 \times 4) + 3 = 11$, I try again the $(3 \times 4) + 3 = 15$, because I am still doubt I try this one $(4 \times 4) + 3 = 19$, $15 + 4 = 19$. So that is my thinking pattern.

Rumus: Gambar ke-1 = $(1 \times 4) + 3 = 7$ persegi
 Gambar ke-2 = $(2 \times 4) + 3 = 11$ persegi
 Gambar ke-3 = $(3 \times 4) + 3 = 15$ persegi
 Gambar ke-n = $(n \times 4) + 3$

Jadi, rumusnya adalah gambar ke-n = $(n \times 4) + 3$

Formula: 1st figure = $(1 \times 4) + 3 = 7$ squares
 2nd figure = $(2 \times 4) + 3 = 11$ squares
 3rd figure = $(3 \times 4) + 3 = 15$ squares
 nth figure = $(n \times 4) + 3$
 So, the formula is the nth figure is $(n \times 4) + 3$

Figure 8. S₂ subject work result

S₂ subject justifying the generalization with the aim of convincing others that the resulting conjecture is correct with a particular example. S₂ points out, from this example, S₂ justifying the generalization results in. This is shown by the following interview quotation.

P 14 : Okay, then how do you explain to others that the resulting formula is correct?

S₂ 14: I will show the results at the 1st, 2nd, 3rd figures and so on. This is the proof (while pointing out the work result). I have tried it continuously, then it is correct. So I believe in the formula.

From the data described based on the conjecturing process steps, it describes S₂ subject thinking structure which is analyzed based on the APOS step. S₂ Conjecturing process in the generalization of pattern problem solving begins by observing case the action step, and organizing the case, then S₂ internalizes the action into prose by finding and predicting the pattern. Once internalized into the action, S₂ encapsulates the action into the by formulating the conjecture and validating the conjecture. Then at the following schema step, S₂ Generalizing the conjecture and justifying the resulting conjecture. S₂ thinking structure is presented in Figure 9.

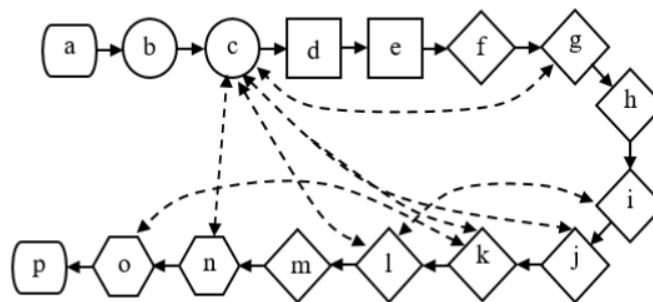


Figure 9. S₂ subject thinking structure

Notes:

a : The problem proposed is to find the common formula to set the number of square at n th figure	m : The n th formula is $(n \times 4) + 3$
b : Counting and observing the number of square at the 1 st , 2 nd , and 3 rd figures.	n : Believing in that The n th formula is $(n \times 4) + 3$
c : Counting the number of square at the 1 st , 2 nd ,	o : Validating the n th formula by specific case

and 3 rd figures	
d : Making list of table to sort the pattern of 7, 11, 15	p : Done
e : Counting the square different at the 1 st , 2 nd , and 3 rd figures and thinking of the following object	→ Activity sequence
f : Stating the row different is 4	↔ Validation activity, for example from g to c, then go back to g; from l to i, then go back to l, etc
g : Looking for suitable n to sort the 1 st , 2 nd , and 3 rd figures	○ Action
h : Supposed, $7 + 4 = 11$, if $n + 4$ it can not be	□ Process
i : Trying to input 1 st (one) figure into the formula because one is n	◇ Object
j : Trying (1×1) adds with what number to be 7. $(1 \times 1) + 6 = 7$ correct, $(2 \times 1) + 6 = 8$ incorrect	○ Schema
k : Trying (1×2) adds with what number to be 7. $(1 \times 2) + 5 = 7$ correct, $(2 \times 2) + 5 = 9$ incorrect	
l : Trying (1×4) adds with what number to be 7. $(1 \times 4) + 3 = 7$ correct, $(2 \times 4) + 3 = 11$ correct $(3 \times 4) + 3 = 15$ correct, $(4 \times 4) + 3 = 19$ correct	□ Initial and final activities

3.3 Global Conjecturing Process Schema of S1 subject and S2 subject in Generalization of Pattern Problem Solving based on APOS

In generalizing [16] the patterns, S_1 and S_2 have been aware that the 1st figure, 2nd figure and 3rd figure form a pattern. To find a common formula of the number of square at the n^{th} Figure At this action step, S_1 and S_2 observe and count the number of square regardless the black square and white one, at the 1st figure, 2nd figure and 3rd Figure Based on the number of square at the 1st figure, 2nd figure and 3rd figure, S_1 organizes cases by sorting the number row patterns and S_2 registers to relate number one with the 1st figure, number 2 with 2nd figure, number 3 with the 3rd figure, and so on. Then, at the process step, S_1 and S_2 are searching for and predicting the pattern by looking at the difference between the 2nd figure and the 1st figure, 3rd figure and the 2nd figure and think that the following figure increases by 4.

The object step, to formulate conjecture, S_1 sees the 1st figure addition to the 2nd figure is 4, and the 2nd figure to the 3rd figure is also 4, by looking at the addition, S_1 looks for the initial number before adding by 4 times n . For the 2nd figure $x + (4 \times 2) = 11$, then $x = 3$, for the 3rd figure $x + (4 \times 3) = 15$ then $x = 3$ so the initial number before added 4 times n is 3. S_2 Subject tries to determine the suitable n based on the figure sequence, then S_2 tries to enter the 1st (one) figure into the formula because one is n , by trying one by one starting from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$. The conjecture produced by S_1 and S_2 is $3 + (4 \times n)$ and validate the conjecture based on specific examples obtained at the action or process step.

The scheme step, S_1 and S_2 subjects generalize the conjecture to believe that the conjecture resulted is correct after validating the conjecture in the previous steps. In justifying the generalization with the aim of convincing others that the resulting conjecture is correct, S_1 and S_2 use specific examples obtained at the action step or process step. Justifying the generalizations made by the subject

S_1 and S_2 is the same as what has done at the validation step namely using the specific examples. The Global conjecturing process scheme of S_1 subject and S_2 subject in the solving of pattern generalization problem is presented in Figure 10.

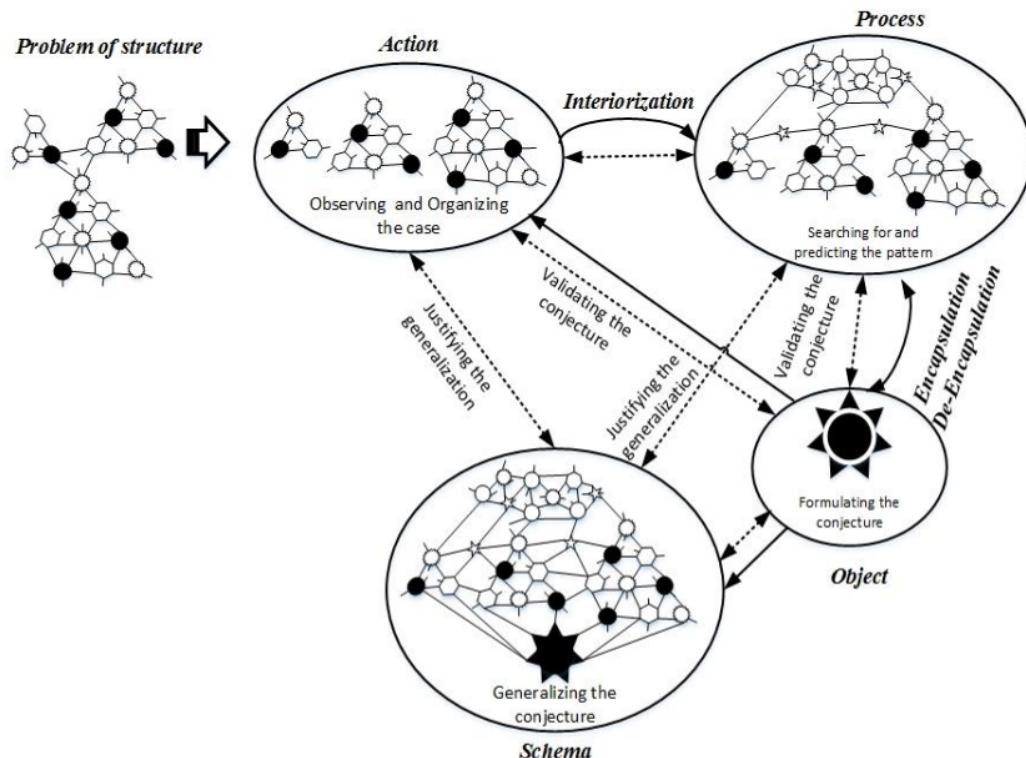


Figure 10.The schema of global conjecturing process

4 Discussion

This section will discuss the research findings related to global conjecturing process to solve problem of pattern generalization. In generalizing the pattern, S_1 and S_2 on the "action" step have realized that the 1st figure, 2nd figure, and 3rd figure form a pattern. To find a common formula number of square in the nth, S_1 and S_2 , it observed by counting number of square regardless the black and white squares on the 1st figure, 2nd figure, and 3rd figures. Based on the number of square at the 1st figure, 2nd figure, and 3rd figure, S_1 organizes the case by sorting the row pattern of 7, 11, 15 and so on while S_2 makes a list or a table to relate number one with the 1st figures, number 2 with the 2nd figure, number 3 with the 3rd figure. This shows that at the "action" step, S_1 subject and S_2 subject observe and organizes the cases regardless the black and white squares, therefore the conjecturing process conducted by the subjects is referred to as the global conjecturing process. Observing cases and organizing cases regardless the black and white squares are based on the Gestalt laws in observation, called similarity Law which is a person tends to perceive the same holistic stimulus [27].

This "process" step, the subjects internalize the action to find and predict the pattern by investigating the distinguishing between number of square at the 2nd, 1st, 3rd, and 2nd figures. and think that the following figure has the same pattern, namely obtaining the increased 4. In formulating the conjecture step, S_1 conducts the encapsulation to generate the object which is to see the 1st figure

addition to the 2nd figure is 4, and the 2nd figure to 3rd figure is also 4. S₁ seeks the initial number before adding 4 times n. for the 2nd figure, $-2x + (4 \times 2) = 11$ so $x = 3$, for the 3rd figure $-3x + (4 \times 3) = 15$ so $x = 3$ so the initial number before being added 4 times n is 3. After finding the initial number, S₁ formulate the common formula of the conjecture namely $3 + (4 \times n)$ and validating the conjecture based on the number of square known. S₂ tries to det the appropriate n based on the figure sequence, after that S₂ tries to enter the 1st figure into the formula because one is also n, by trying one by trying one by one sta₄ng from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$ then S₂formulates the common formula of th₁₂ conjecture to set the number of square at the nth figure $n = (n \times 4) + 3$ and validate the conjecture based on the number of squares on the 4th figure and 5th figure. The way done by S₁ is looking for the initial number using the x symbol and S₂ seeks the appropriate based on the figure sequence. Both ways are different but meaningful for itself to find the common formula of the conjecture, it describes the knowledge possessed. This is consistent to what expressed by[28]hat the mathematical symbol is a tool for coding and describing the knowledge as well as communicating the mathematical knowledge. At this process step and object step, the subjects conduct 29 perfectly.

At this scheme step, S₁ subject and S₂ subject generalize the conjecture to believe that the resulting conjecture is correct. In justifying the generalization with the aim of convincing others that the resulting conjecture is correct, S₁ and S₂ use the specific examples obtained at the action step and object step. S₁ justifying the generalization with the aim of convincing others that the resulting conjecture is correct with the specific example. S₂ counts the number of square at the 4th figure namely $n = 3 + (4 \times 4) = 19$ and 19 is also obtained from 3rd figure plus 4 is 19, from this example, S₁ validates the resulting generalization. S₂ validates the generalization with aim of convincing others that the resulting conjecture is correct with specific example obtained at the object step by pointing out $(1 \times 4) + 3 = 7$, $(2 \times 4) + 3 = 11$, $(3 \times 4) + 3 = 15$, and $(4 \times 4) + 3 = 19$. In justifying the generalizations, S₁ subject and S₂ subject conduct their own way, this is based on the [3] that the students do not j₁₀ simply use the notation or symbols but also their presentation and give a reason mathematically, make conclusions and generalizations in their way. At this scheme step, it is also conducted perfectly.

5 Conclusions

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Based on the findings in the global conjecturing process conducted by the students in the gene₃zation of pattern problem solving, it increases the conjecturing process theories [4] about the type of empirical induction from a finite number of discrete cases which has seven steps and n₇ study the students' thinking process in constructing the conjecture generalization. The results show that the global conjecturing process occurs at the step of action₇ in which subjects build a conjecture by observing and counting the number of squares complete, at the step of process, the object and scheme were perfectly performed.

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References

- [1] Bergqvist T 2005 How students verify conjectures: teachers' expectations. *Journal of Mathematics Teacher Education* **8** 171–191
- [2] Sutarto and Hastuti I D 2015 Conjecturing Dalam Pemecahan Masalah Generalisasi Pola. *Jurnal Ilmiah Mandala Education* **1** 2 172-178
- [3] Cañadas M C and Castro E 2005 A proposal of categorisation for analysing inductive reasoning *In M. Bosch (Ed.), Proceedings of the CERME 4 International Conference* 401-408
- [4] Cañadas M C, Deulofeu J, Figueiras L, Reid D A and Yevdokimov O 2007 The conjecturing process: Perspectives in theory and implications in practice *Journal of Teaching and Learning*, 1 55–72

- [5] Dindyal J 2007 High school students' use of patterns and generalisations *In J. Watson & K. Beswick (Eds), Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia* 236-245
- [6] Dubinsky E 2001 Using a theory of learning in college *TaLUM* **12** 10 – 15
- [7] Dubinsky E and Mc Donald M 2011 APOS: A constructivist theory of learning in undergraduate mathematics education research *In D. Holton et al. (Eds), The teaching and learning of mathematics at university level: An ICMI Study* 273-280
- [8] Fischbein E 2002 *Intuition in science and mathematics* An Educational Approach NewYork: Kluwer Academic Publisher.
- [9] Furinghetti F and Paola D 2003 To produce conjectures and to prove them within a dynamic geometry environment: A case study *In N. A. Pateman, B. J. Doherty, & J. Zilliox (Eds.), Proceedings of the Twenty-Seventh Annual Conference of the International Group for the Psychology of Mathematics Education* 397-404
- [10] Küchemann D 2010 Using patterns generically to see structure *Pedagogies: An International Journal* **5** 3 233–250
- [11] Hastuti I D, Nusantara T and Susanto H 2016 Constructive metacognitive activity shift in mathematical problem solving *Educational Research and Reviews* **11** 8 656
- [12] Lakatos I 2015 *Proofs and refutations: The logic of mathematical discovery* Cambridge university press
- [13] Lee K H and Sriraman B 2010 Conjecturing via reconceived classical analogy *Educational Studies in Mathematics* **76** 2 123–140
- [14] Lin F L 2006 Designing mathematics conjecturing activities to foster thinking and constructing actively. *APEC-TSUKUBA International Conference*, Tsukuba, Japan
- [15] Mason J 2002 Generalization and algebra: Exploiting Children's Powers *In L. Haggerty (Ed.), Aspects of Teaching Secondary Mathematics: Perspectives on Practice* 105-120
- [16] Mason J, Burton L and Stacey K 2010 *Thinking mathematically second edition*, England: Pearson Education Limited
- [17] Mulligan J and Mitchelmore M 2009 Awareness of pattern and structure in early mathematical development *Mathematics Education Research Journal* **21** 2 33-49
- [18] Mulligan J T, Mitchelmore MC, English LD and Robertson G 2011 Implementing a Pattern and Structure Mathematics Awareness Program (PASMAP) In Kindergarten *In L. Sparrow, B. Kissane, & C. Hurst (Eds.) Shaping the Future of Mathematics Education. Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia* 797–804
- [19] Nasional Council of Teacher of Mathematics 2000 *Principles and standards for school mathematics* Reston, VA: NCTM
- [20] Pólya G 1967 *Le découverte des mathématiques* París: DUNOD
- [21] Reid D 2002 Conjectures and refutations in grade 5 mathematics *Journal of Research Mathematics and Education* **33** 1 5-29
- [22] Resnik M D 2005 *Mathematics as a science of mathematics* Oxford: University Press
- [23] Sutarto Toto N and Subanji S 2015 Indicator of conjecturing process in a problem solving of the pattern generalization *In Proceeding International Conference on Educational Research and Development (ICERD), UNESA Surabaya* 32-45
- [24] Sutarto T N and Subanji S 2016 Local conjecturing process in the solving of pattern generalization problem *Educational Research and Reviews* **11** 8 732
- [25] Tikekar V G 2009 Deceptive patterns in mathematics *International Journal of Mathematical Science Education* **2** 1 13-21
- [26] Vogel R 2005 Patterns: A fundamental idea of mathematical thinking and learning *ZDM* **37** 5 445-449
- [27] King D B and Wertheimer M 2009 *Max Wertheimer & Gestalt Theory* New Brunswik, New Jersey: Transaction Publisher

- [28] Schwartz J L, Yerushalmy M and Wilson B *The geometric supposer: what is it a case of?* Hillsdale, NJ Lawrence Erlbaum Associates
- [29] Zazkis R and Liljedahl P 2002 Generalization of patterns: the tension between algebraic thinking and algebraic notation *Educational Studies in Mathematics* **49** 379–402
- [30] Hastuti I D 2016 Pergeseran aktivitas metakognitif siswa dalam pemecahan masalah matematika Disertasi tidak diterbitkan Malang: PP UM
- [31] Sutarto T N and Subanji S 2016 Local conjecturing process in the solving of pattern generalization problem *Educational Research and Reviews* **11** 8 732

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