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Global conjecturing process in pattern generalization problem

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Abstract. The aim of this global conjecturing process based on the theory of APOS. The subjects used in study were 15 of 8th grade students of Junior High School. The data were collected using Pattern Generalization Problem (PGP) and interviews. After students had already completed PGP; moreover, they were interviewed using students work-based to understand the conjecturing process. These interviews were video taped. The result of study reveals that the global conjecturing process occurs at the phase of action in which subjects build a conjecture by observing and counting the number of squares completely without distinguishing between black or white squares, finaly at the phase of process, the object and scheme were perfectly performed.

1. Introduction

One of the standard assessments used in this study begins from preschool until secendary school level [19] is making and examining the mathematic conjecture. Furthermore, it is explained that making conjecture is important work because it functions as the basic to develop and increase new perception for further study. Making and examining conjecture is a step in mathematical study [12], reasoning [21], and mathematical thinking [16].

Conjecture is the logical statement where its truth is not yet certain [3, 8, 16, 21]. Along with this idea, conjecture is a statement concerning all the possible cases, based on the empirical facts, but with the doubtful elements [4]. Based on these arguments, it can be argued that conjecture is the statement based on the reasoning process where its truth is not yet certain.

Conjecture and problem solving are linked activity. Conjecture and problem solving are the important parts and interrelated in the mathematic activity [4, 2] Moreover, it is said that the problem solving involves the finding whiler conjecture is the main road for the finding [19]. In problem solving, conjecture helps the problem solver to find the solution for the problems faced. As a result, conjecture does not just show up, but there is a process and the process is the conjecturing process.

The conjecturing process is the process of constructing the conjecture [4,16]. Assumes that the conjecturing process is the mental activity expressed in problem solving based on the knowledge which has been owned and the trust is necessary to be proven [8]. Based on the argument, it can be concluded that the conjecturing process is the mental activity in constructing conjecture based on the possessed knowledge. The mental activity is the process in the mind which can be seen ins the students' behavior in problem solving [11, 30, 31].

There will be the conjecturing process if the students face any problems. In the conjecturing process, the students construct the conjecture based on their knowledge. There is no validation in the conjecture built by the students. It depends on the students' involment. If there is a validation for the conjecture they built, the conjecture is considered as the correct one. Only after the conjecture is validated then one can consider it as correct or incorrect. If the conjecture does not have incorrect value the process of conjecturing will be processed again until the it has the correct value. The conjecture with the correct value is the solution for the problems faced by the students.

In relation to conjecturing process, the familiar one used is mathematic problem solving as it is the conjecturing type of empirical induction from a finite number of discrete cases [4]. This type of conjecturing process consists of seven steps namely observing the case, organizing the case, finding and predicting the pattern, formulating the conjecture, validating the conjecture, generalizing the conjecture, and validating the generalization. The conjecturing process of this type—by inducting from a finite number of discrete cases—is mostly found in the problem related to the numbers, where the pattern under observation is consistent. In the problem solving involving the numbers with consistent pattern, the seven conjecturing processes do not always take place; there are many factors affecting process such as the type of task or the students' characteristics involved [3].

The pattern described as the regularity which can be predicted above commonly involves the numerical, spatial or logical relations [17]. Many mathematicians state that the mathematics is a 'science about pattern' [22, 25]. They highlight the pattern existence in all mathematic fields [25]. In particular the pattern is considered by some researchers as the strategy applied in algebraic field because the pattern is the basic measure to construct the generalization which is the mathematic essence [29].

Generalization of pattern is an important aspect in school mathematical activities [5, 18, 26, 29]. Along with this idea, the generalization must be the core of the school mathematical activities [10]. The generalization of pattern itself is the activity making the pattern common rule based on some special examples. The common rule obtained is the conjecture, and the generalization is the specific type of conjecture obtained from common reasoning [28].

The significant contribution to the conjecture or conjecturing has been studied by researchers who consider the conjecture as the intuition of expression [8]. It shows the important of conjecturing atmosphere [15]. Research has been carried out on the production of conjectures within a dynamic geometry environment [9] by analyzing how the students verify the conjecture and how the teachers' trust related to this process [1]. Researchers developed an open classical analogy in geometry construction [13] by designing mathematic conjecturing activities to foster thinking and constructing actively [14]. Then, they analyze various familiar types and steps of conjecturing process in problem solving [4]. Among the studies, it is not revealed yet how the students—through conjecturing process—generalizing the pattern of problem solving.

The conjecturing process described above corelates to some theories [3, 4, 20, 21, 23]. These theories are the basic of conjecturing process of empirical induction from a finite number of discrete cases. They consist of four inductive reasoning processes in problem solving, namely (1) observing specific cases, (2) formulating the conjecture based on previous case, (3) generalization, and (4) conjecture verification with specific new cases [20]. Reid uses the inductive reasoning process in the context of empirical induction from a finite number of discrete cases by the steps: (1) observing specific cases, (2) observing the pattern, (3) formulating the conjecture for common cases (with doubtfulness), (4) generalization, and (5) using generalization to prove [21]. there are seven steps in describing the inductive reasoning process, namely (1) Observing cases, (2) Organizing cases, (3) Searching for and predicting patterns, (4) Formulating a conjecture, (5) Validating the conjecture, (6) Generalizing the conjecture, (7) Justifying the generalization [3].

There are seven steps in describing the inductive reasoning process drive from Canada's, as such a types of conjecturing process namely empirical induction from finite number of discrete cases. The conjecturing process in this study is empirical induction from a finite number of discrete cases. The explanation of those seven steps take place in conjecturing process [24]. Based on its explanation and indicator adapted from the students. the conjecturing process in generalization of pattern problem solving is grouped into two types, i.e. global conjecturing, and local conjecturing. The global conjecturing is the mental activity in constructing the conjecture by observing the problems intact, and The local conjecturing is the mental activity of constructing the conjecture by observing the problems separately. While, The global conjecturing process is often done by the students in generalization of pattern problem solving. Thus, this study will describe the global conjecturing process based on APOS theory.

The mental activity in conjecture analyzes using APOS theory, because APOS theory is a theory that can be used as the analytical tool to describe one's developmental scheme in a mathematic topic--as it is the totality of the related knowledge (aware or unaware) to the topic [6]. This theory is based on the hypothesis of mathematic knowledge that may use to solve the situation as the mathematical problem by constructing the action, process and object as well as regulating the scheme to comprehend the situation and solve the problem [7]. This theory is called APOS and use to describe an action at the interiorization as the Process. The Process is encapsulated in an object. Then, it is related to other knowledge in a schema. A schema can also be encapsulated as an object. Problem of Research: How is the global conjecturing process in the solving of pattern generalization problem based on APOS theory?

2. Methodology of Research

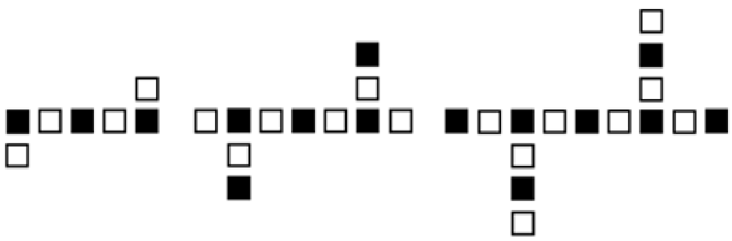
2.1. Subjects

The subjects in this study are 15 students of class VIII derived from 9 students of VIII State Junior High School 1 Malang, and 6 students of State Junior High School 3 Malang.

2.2. Instrument

There are two types of instruments use. The main instrument is the researchers themselves who act as planners, data collectors, data analysts, interpreters, and reporters of research results. The auxiliary instrument used in this study is a Pattern Generalization Problem (PGP) and interviews. The problem given aims to obtain a description of the process of conjecturing of the students, while the interview used was unstructured interview. The PGP is presented in Figure 1.

Pay attention to the square pattern forms by the 1st figure, 2nd figure and 3rd figure below!



1st Figure 2nd Figure 3rd Figure

Find out the common formula to set the number of square at the nth figure from the formed pattern! Explain how do you find the common formula!

Figure 1. The Pattern Generalization Problem (PGP)

2.3. Data Analysis

This study is a qualitative research with descriptive exploratory approach. At the data analysis step, the activities conducted by researchers were (1) transcribing the data obtained from interviews, (2) data condensation, including explaining, choosing principal matters, focusing on important things, removing the unnecessary ones, and organizing raw data obtained from the field (3) encoding the data from PGP answer sheet and interviews refer based on indicators of local conjecturing process are presented in Table 1, (4) describing the global conjecturing process in the solving of pattern generalization problem based on APOS theory, and (5) conclusion.

3. Results of Research

Based on the analysis results of the answer sheets and the interview results, it is obtained the data on the global conjecturing process conducted by the students in the generalization of pattern problem solving based on the APOS theory. After getting bored for the subject taking process, it is obtained 6 subjects who conduct the global conjecturing process, 5 subjects who conduct the contrast conjecturing and 3 subjects who conduct the local conjecturing and generalizing symbolic. Out of 6 subjects, it will be described two subjects making the global conjecturing process of generalization of pattern problem-solving which is the S1 subject and S2 subject. The data presented is obtained by the procedures (1) the subjects complete the PGP, and (2) after the subjects complete the PGP, then they are interviewed to explore about the global conjecturing process which has been conducted. The data presentation and analysis of the global conjecturing processes in the generalization of pattern problem solving is as the following.

3.1. S1 subject data presentation

In generalizing the S_1 patterns, it has realized that 1^{st} figure, 2^{nd} figure, and 3^{rd} figure form a pattern. To find a common formula on the number of square at the n^{th} figure, S_1 observes and counts the number of square regardless the black square and white ones at the 1^{st} figure, 2^{nd} figure, and 3^{rd}Figure Here are the interview quotation and the S_1 work results in completing the following PGP.

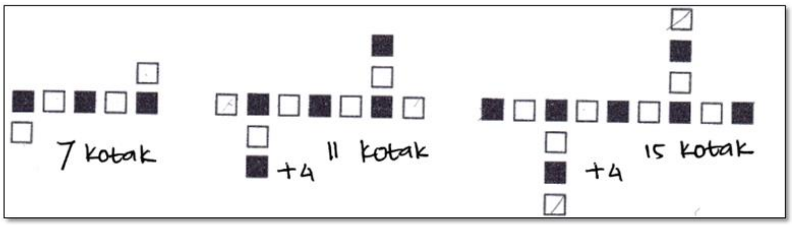


Figure 2.S_1 subject work result

$S_1$04 : this is the different of the figure, Sir. This is the first, 7 and the second one is 11, and the third one, the number is 15 (while pointing to the square figure). The different is 4, so the following figure is plus 4, plus 4, plus 4.

Based on the number of square in the 1^{st} figure, 2^{nd} figure, and 3^{rd} figure, S_1 organizes the cases by ordering the number row pattern. Then, S_1 finds and predicts the pattern by seeing the different between the 2^{nd} figure and the 2^{nd} figure, the 3^{rd} figure and the 2^{nd} figure and thinking how the following figure is plus 4, plus 4, plus 4. This is confirmed by the $S_1$04 interview quotation and the students' work results in completing the following PGP.

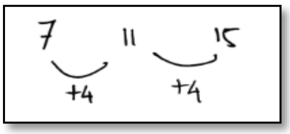


Figure 3. S_1 subject work result

To formulate the conjecture, S_1 subject sees the addition of 1^{st} figure into the 2^{nd} figure is 2, and the 2^{nd} figure into the 3^{rd} figure is also 4, by seeing this addition, S_1 formulates the n^{th} formula conjecture in setting the number of square at the figure is $n = n + 4$. After that, S_1 validates the conjecture by seeing the appropriateness with the number of square at the 4^{th} figure and 3^{rd} figure, then saying that the n^{th} formula is incorrect. The following is the S_1 interview quotation.

$S_1$08 : this adds by 4, then adds by 4, then adds by 4. But, thinking it continuously, it may be incorrect. It is why, when 4 adds by 4, it is 8, then when n is 3, it adds by 4 the result is only 7. So it is incorrect.

After realizing that the conjecture formulated is incorrect, S_1 tries new strategy to formulate the conjecture namely by finding the initial number before it is plus 4 because the pattern always adds by 4. S1 finds the initial number by looking for the different of the number of the square at the 1st figure, 2nd figure and 3rd figure by 4. The subtraction results consecutively are 3, 7, 11. S1 realizes that the initial number searched is not yet correct because its initial number is still different. This is shown by the interview quotation and the S_1 work results as the following.

$S_1$11 : this is initially still different (3, 7, 11) meaning that it is incorrect (while pointing out the work result)

$$\begin{array}{lll} 7 = n+4 & 11 = n+4 & 15 = n+4 \\ 7-4 = n & 11-4 = n & 15-4 = n \\ 3 = n & 7 = n & 11 = n \end{array}$$

Figure 4. S_1 subject work result

S_1 subject then uses the new strategy which is to look for the initial numbers before adding by 4 times n. S1 writes down the initial number symbols with x, for the 2nd figure $x + (4 \times 2) = 11$, then $x = 3$, for the 3rd figure$x + (4 \times 3) = 15$ then $x = 3$ so the initial number before plus 4 times n is 3. After finding the initial figure, S_1 formulates the conjecture namely the common formula which is $3 + (4 \times n)$and validates the conjecture based on the number of squares which are already known. After validating S_1 generalizes the conjecture as to believe that the common formula is $3 + (4 \times n)$It is also shown from the quotation interview and S_1 work result as the following.

$S_1$11 : I look for the initial number before it is plus 4 times n. the second figure is the same to $x + (4 \times 2) = 11$ so $x = 3$. Then the 3rd figure is similar to $x + (4 \times 3) = 15$ so $x = 3$. So, the intial number before it is plus 4 times n is 3.
P 13 : Okay, then are you sure by the common formula you obtain?
$S_1$13 : yes, Sir I am…

$$\begin{array}{ll} G_2 = x + (4 \times n) & G_3 = x + (4 \times n) \\ 11 = x + (4 \times 2) & 15 = x + (4 \times 3) \\ 11 = x + 8 & 15 = x + 12 \\ 11 - 8 = x & 15 - 12 = x \\ 3 = x & 3 = x \\ n = 3 + (4 \times n) & \end{array}$$

Figure 5. S_1 subject work result

S_1justifying the generalization with the aim to convince others that the conjecture obtained is correct with a particular example. S_1 describes how to obtain the formula, and shows an example of the formula suitability with a number for a square at the 1st, 2nd, 3rd figures and calculate the number of square at the 4th figure like what has done at the validation step which $n = 3 + (4 \times 4) = 19$ and 19 is also obtained from the 3rd figure plus 4 is 19, from this example S_1 justifying the resulting generalizations. This is shown by the interview quotation as the following.

P 19 : Okay, then how do you explain that the resulting *formula is correct?*
$S_1$19: I will explain how I get the formula and show the example for the 1st, 2nd, 3rd and 4th figures. For example, for the 4th figure, $n = 3 + (4 \times 4) = 19$. *19 is also obtained from the 3rd fig.,* $15 + 4 = 19$. *(while pointing put the work). s*

From the data described based on the conjecturing process steps, it can be described the S_1 subject thinking structure analyzed based on the APOS step. The S1 conjecturing process in the generalization of pattern problem solving begins with the action steps namely observing case, and organize the case, then S_1 internalizes the action into prose by finding and predicting the pattern. Once internalized the action into the process, S_1 encapsulates the process into the object by formulating the conjecture and validating the conjecture. At the following scheme step, S_1 generalizes the conjecture and justifying the resulting conjecture. S_1 thinking structure is presented in Figure 6.

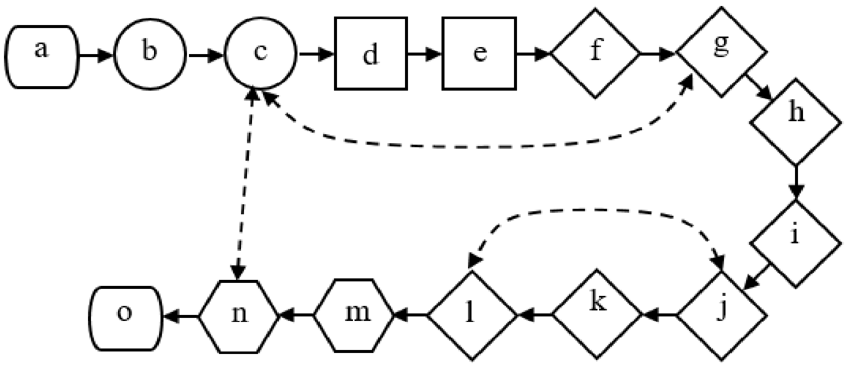


Figure 6. S_1 subject thinking structure

Notes:

a :	The problem proposed is to find the common formula to set the number of square at nth figure	l :	The nth formula is $3 + (4 \times n)$
b :	Counting and observing the number of square at the 1st, 2nd, and 3rd figures.	m :	Believing in that The nth formula is $4n + 3$
c :	Counting the number of square at the 1st, 2nd, and 3rd figures	n :	Validating the nth formula by pointing out at the example at the 1st, 2nd, 3rd, and 4th figures, supposed the 4th figure $n = 3 + (4 \times 4) = 19$. 19 is also obtained from the 3rd figure, $15 + 4 = 19$.
d :	Writing down the row pattern of 7, 11, 15	o :	Done
e :	Counting the square different at the 1st, 2nd, and 3rd figures and thinking of the following object		Activity sequence
f :	Stating the row different is 4		Validation activity, for example from g to c, then go back to g; from l to i, then go back to l, etc
g :	The addition is 4 so $n = n + 4$		Action
h :	Finding the initial number before being		Process

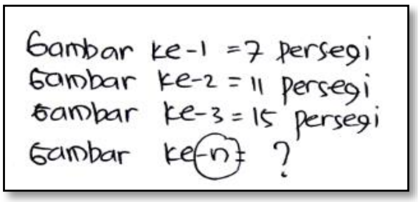
	added by 4.		
i :	Counting the different of number of square at the 1^{st}, 2^{nd}, 3^{rd} and 4^{th} figures, the results are 3, 7, 9.	◇	Object
j :	$\mathrm{x} + (4 \times 2) = 11, \mathrm{x} = 3$ $\mathrm{x} + (4 \times 3) = 15, \mathrm{x} = 3$	⬡	Schema
k :	The initial number before 4 times n is 3	▢	Initial and final activities

3.2 S_2 subject data presentation

In generalizing the patterns, S_2 subject has been aware that the 1^{st} figure, 2^{nd} figure and 3^{rd} figure form a pattern. To find a common formula of the number of square at the n^{th} figure, S_2 observes and counts the number of square regardless the black square and white one at 1^{st} figure, 2^{nd} figure and 3^{rd}Figure Here is the interview quotation of S_2.

P 04 : what do you think first when reading this problem?
$S_2$04 : at first I look at the figure, then from this figure, I look at another figure continuously, then it compare both (while pointing out at the PGP)
P 05 : Then you compare, what does it mean?
$S_2$05 : ...When comparing both, I find if in each figure there is 4 addition, four square addition. I still can not see the white and black. I don't see it. Then at first, I think of this continuously, the pattern is always like this(while pointing out at the PGP)

Based on the number of square at the that the 1^{st} figure, 2^{nd} figure and 3^{rd} figure, S_2 subject organizes cases by signing up to number one with the 1^{st} figure, number two with the 2^{nd} figure, number three with the 3^{rd} figure, and so on. Furthermore, S_2 locates and predicts the patterns by comparing the number of squares at the 1^{st} figure, 2^{nd} figure and 3^{rd} figure and finds that the number of additional figure is always 4 and thinks of that the pattern always continues. This is confirmed by $S_2$05 interview quotation and the student's work results in completing the following generalization of pattern problem solving.



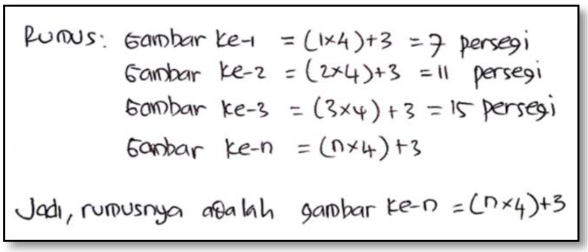
1^{st} figure = 7 squares
2^{nd} figure = 11 squares
3^{rd} figure = 15 squares
N^{th} figure = ?

Figure 7. S_2 subject work result

To formulate a conjecture S_2 tries to determine suitable n based on the figure sequence. For example, 7 squares and 11 squares, this means that it has to plus 4, if n then n + 4 it can not be. So it must determine a suitable n based on the figure sequence. After S_2 tries to enter 1^{st} figure (one) into the formula because one is also n, by trying one by one starting from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$. Then, S_2 formulates a common formula of conjecture to determine the number of square at the n^{th} figure $= (n \times 4) + 3$ and validates the conjecture based on the number of squares at the 4^{th} figure and 5^{th}Figure After validating, S1 generalizes the conjecture as to believe that the common formula is $= (n \times 4) + 3$. It is also shown from the interview quotation and the students' work in completing the following generalization of pattern problem solving.

$S_2$08 : so, I try the first one, supposed 1×1, 1×1 must be added with what number to be 7, eee.. then it is plus 6, but if it is supposed $2 \times 1 + 6$ then the results is not 11, so I keep trying the

closest ine which is the most appropriate answer for this square ((while pointing out at the square figure at PGP). I keep trying $(1 \times 2) + 5 = 7$ *is correct, then this one* $(2 \times 2) + 5$ *the result is 9 and not 11, so I keep trying four* $(1 \times 4) + 3 =$ *this 7, I try if it is* $(2 \times 4) + 3 = 11$*, I try again the* $(3 \times 4) + 3 = 15$*, because Iam still doubt I try this one* $(4 \times 4) + 3 = 19$*,* $15 + 4 = 19$*. So that is my thinking pattern.*



Formula : 1^{st} figure = (1 x 4) + 3 = 7 squares
2^{nd} figure = (2 x 4) + 3 = 11 squares
3^{rd} figure = (3 x 4) + 3 = 15 squares
n^{th} figure = (n x 4) + 3
So, the formula is the n^{th} figure is (n x 4) + 3

Figure 8. S_2 subject work result

S_2 subject justifying the generalization with the aim of convincing others that the resulting conjecture is correct with a particular example. S_2 points out, from this example, S_2 justifying the generalization results in. This is shown by the following interview quotation.

P 14 : Okay, then how do you explain to others that the resulting formula is correct?
$S_2$14: I will show the results at the 1^{st}, 2^{nd}, 3^{rd} figures and so on. This is the proof (while pointing out the work result). I have tried it continuously, then it is correct. So I believe in the formula.

From the data described based on the conjecturing process steps, it describes S_2 subject thinking structure which is analyzed based on the APOS step. S_2 Conjecturing process in the generalization of pattern problem solving begins by observing case the action step, and organizing the case, then S_2 internalizes the action into prose by finding and predicting the pattern. Once internalized into the action, S_2 encapsulates the action into the by formulating the conjecture and validating the conjecture. Then at the following schema step, S_2 Generalizing the conjecture and justifying the resulting conjecture. S_2 thinking structure is presented in Figure 9.

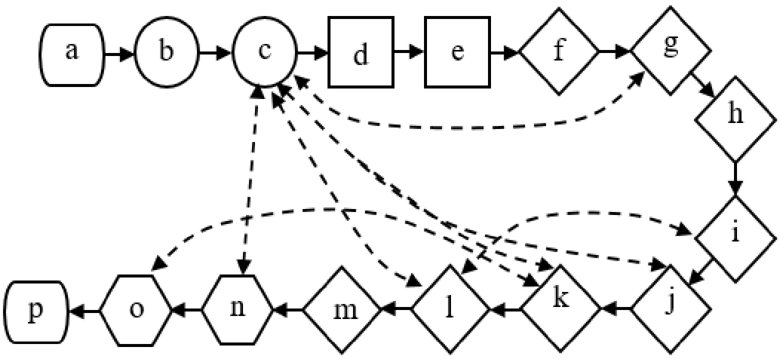


Figure 9. S_2 subject thinking structure

Notes:

a :	The problem proposed is to find the common formula to set the number of square at n^{th} figure	m :	The n^{th} formula is $(n \times 4) + 3$
b :	Counting and observing the number of square at the 1^{st}, 2^{nd}, and 3^{rd} figures.	n :	Believing in that The n^{th} formula is $(n \times 4) + 3$
c :	Counting the number of square at the 1^{st}, 2^{nd},	o :	Validating the n^{th} formula by specific case

	and 3rd figures		
d :	Making list of table to sort the pattern of 7, 11, 15	p :	Done
e :	Counting the square different at the 1st, 2nd, and 3rd figures and thinking of the following object	→	Activity sequence
f :	Stating the row different is 4	◄- -►	Validation activity, for example from g to c, then go back to g; from l to i, then go back to l, etc
g :	Looking for suitable n to sort the 1st, 2nd, and 3rd figures	◯	Action
h :	Supposed, $7 + 4 = 11$, if n + 4 it can not be	□	Process
i :	Trying to input 1st (one) figure into the formula because one is n	◇	Object
j :	Trying (1×1) adds with what number to be 7. $(1 \times 1) + 6 = 7$ correct, $(2 \times 1) + 6 = 8$ incorrect	⬡	Schema
k :	Trying (1×2) adds with what number to be 7. $(1 \times 2) + 5 = 7$ correct, $(2 \times 2) + 5 = 9$ incorrect	▢	Initial and final activities
l :	Trying (1×4) adds with what number to be 7. $(1 \times 4) + 3 = 7$ correct, $(2 \times 4) + 3 = 11$ correct $(3 \times 4) + 3 = 15$ correct, $(4 \times 4) + 3 = 19$ correct		

3.3 Global Conjecturing Process Schema of S1 subject and S2 subject in Generalization of Pattern Problem Solving based on APOS

In generalizing the patterns, S_1 and S_2 have been aware that the 1st figure, 2nd figure and 3rd figure form a pattern. To find a common formula of the number of square at the nth Figure At this action step, S_1 and S_2 observe and count the number of square regardless the black square and white one, at the 1st figure, 2nd figure and 3rdFigure Based on the number of square at the 1st figure, 2nd figure and 3rd figure, S_1 organizes cases by sorting the number row patterns and S_2 registers to relate number one with the 1st figure, number 2 with 2nd figure, number 3 with the 3rd figure, and so on. Then, at the process step, S_1 and S_2 are searching for and predicting the pattern by looking at the difference between the 2nd figure and the 1st figure, 3rd figure and the 2nd figure and think that the following figure increases by 4.

The *object* step, to formulate conjecture, S_1 sees the 1st figure addition to the 2nd figure is 4, and the 2nd figure to the 3rd figure is also 4, by looking at the addition, S_1 looks for the initial number before adding by 4 times n. For the 2nd figure $x + (4 \times 2) = 11$, then $x = 3$, for the 3rd figure $x + (4 \times 3) = 15$ then $x = 3$ so the initial number before added 4 times n is 3. S_2 Subject tries to determine the suitable n based on the figure sequence, then S_2 tries to enter the 1st (one) figure into the formula because one is n, by trying one by one starting from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$. The conjecture produced by S_1 and S_2 is $3 + (4 \times n)$ and validate the conjecture based on specific examples obtained at the action or process step.

The scheme step, S_1 and S_2 subjects generalize the conjecture to believe that the conjecture resulted is correct after validating the conjecture in the previous steps. In justifying the generalization with the aim of convincing others that the resulting conjecture is correct, S_1 and S_2 use specific examples obtained at the action step or process step. Justifying the generalizations made by the subject

S_1 and S_2 is the same as what has done at the validation step namely using the specific examples. The Global conjecturing process scheme of S_1 subject and S_2 subject in the solving of pattern generalization problem is presented in Figure 10.

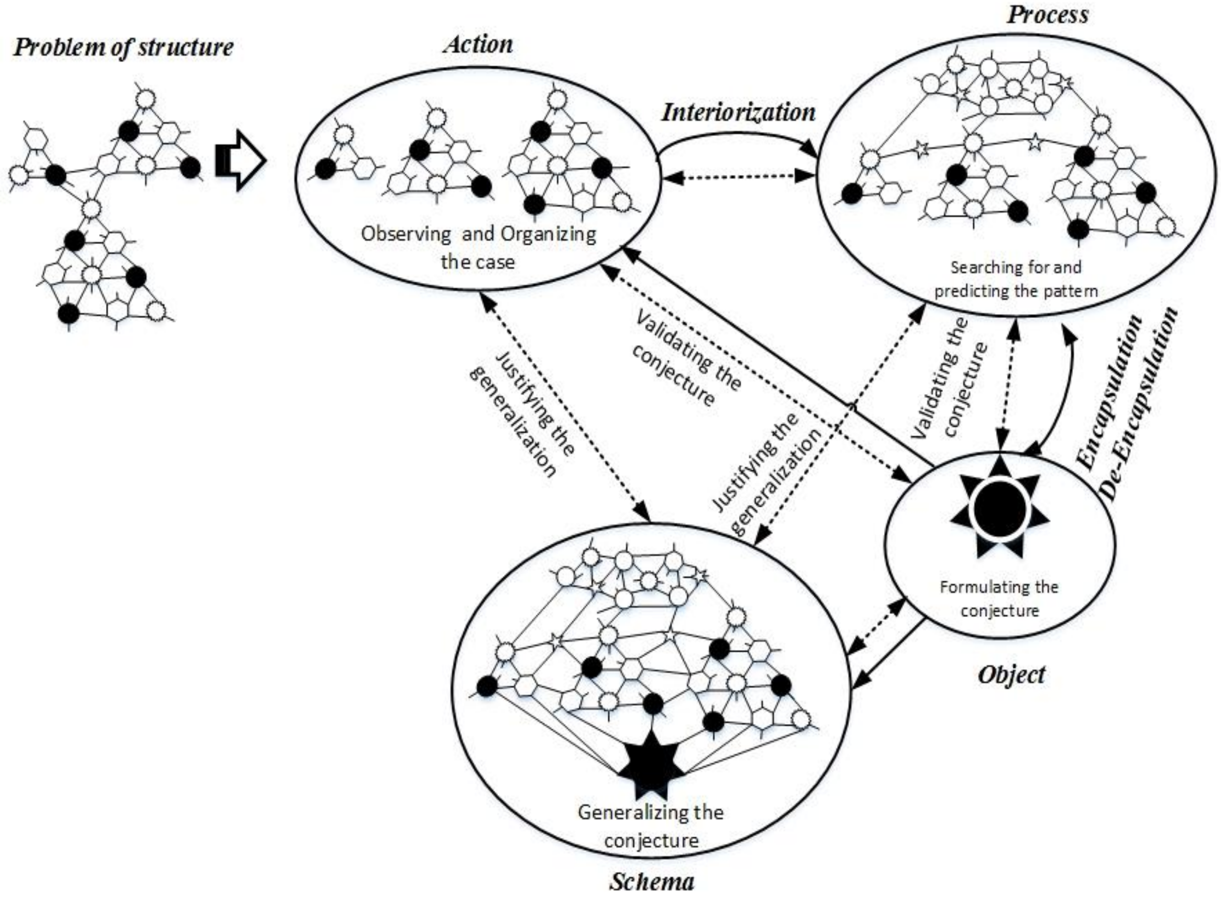


Figure 10. The schema of global conjecturing process

4 Discussion

This section will discuss the research findings related to global conjecturing process to solve problem of pattern generalization. In generalizing the pattern, S_1 and S_2 on the "action" step have realized that the 1st figure, 2nd figure, and 3rd figure form a pattern. To find a common formula number of square in the nth , S_1 and $S_{2,}$, it observed by counting number of square regardless the black and white squares on the 1st figure, 2nd figure, and 3rd figures. Based on the number of square at the 1st figure, 2nd figure, and 3rd figure, S_1 organizes the case by sorting the row pattern of 7, 11, 15 and so on while S_2 makes a list or a table to relate number one with the 1st figures, number 2 with the 2nd figure, number 3 with the 3rd figure. This shows that at the "action" step, S_1 subject and S_2 subject observe and organizes the cases regardless the black and white squares, therefore the conjecturing process conducted by the subjects is referred to as the global conjecturing process. Observing cases and organizing cases regardless the black and white squares are based on the Gestlat laws in observation, called similarity Law which is a person tends to perceive the same holistic stimulus [27].

This "process" step, the subjects internalize the action to find and predict the pattern by inverstigating the distinguish between number of square at the 2nd , 1st , 3rd , and 2nd figures. and think that the following figure has the same pattern, namely obtaining the increased 4. In formulating the conjecture step, S_1 conducts the encapsulation to generate the object which is to see the 1st figure addition to the 2nd figure is 4, and the 2nd figure to 3rd figure is also 4. S_1 seeks the initial number before adding 4 times n. for the 2nd figure, -2 $x + (4 \times 2) = 11$ so $x = 3$, for the 3rd figure -3 $x + (4 \times 3) = 15$ so $x = 3$ so the initial number before being added 4 times n is 3. After finding the initial number, S_1 formulate the common formula of the conjecture namely $3 + (4 \times n)$ and validating the conjecture based on the number of square known. S_2 tries to det the appropriate n based on the figure sequence, after that S_2 tries to enter the 1st figure into the formula because one is also n, by trying one by trying one by one starting from $(1 \times 1) + 6$, $(1 \times 2) + 5$, and $(1 \times 4) + 3$ then S_2formulates the common formula of the conjecture to set the number of square at the nth figure $n = (n \times 4) + 3$ and validate the conjecture based on the number of squares on the 4th figure and 5th figure. The way done by S_1 is looking for the initial number using the x symbol and S_2 seeks the appropriate based on the figure sequence. Both ways are different but meaningful for itself to find the common formula of the conjecture, it describes the knowledge possessed. This is consistent to what expressed by[28]hat the mathematical symbol is a tool for coding and describing the knowledge as well as communicating the mathematical knowledge. At this process step and object step, the subjects conduct it perfectly.

At this scheme step, S_1 subject and S_2 subject generalize the conjecture to believe that the resulting conjecture is correct. In justifying the generalization with the aim of convincing others that the resulting conjecture is correct, S_1 and S_2 use the specific examples obtained at the action step and object step. S_1 justifying the generalization with the aim of convincing others that the resulting conjecture is correct with the specific example. S_2 counts the number of square at the 4th figure namely $n = 3 + (4 \times 4) = 19$ and 19 is also obtained from 3rd figure plus 4 is 19, from this example, S_1 validates the resulting generalization. S_2 validates the generalization with aim of convincing others that the resulting conjecture is correct with specific example obtained at the object step by pointing out $(1 \times 4) + 3 = 7$, $(2 \times 4) + 3 = 11$, $(3 \times 4) + 3 = 15$, and $(4 \times 4) + 3 = 19$. In justifying the generalizations, S_1 subject and S_2 subject conduct their own way, this is based on the [3] that the students do not just simply use the notation or symbols but also their presentation and give a reason mathematically, make conclusions and generalizations in their way. At this scheme step, it is also conducted perfectly.

5 Conclusions

Based on the findings in the global conjecturing process conducted by the students in the generalization of pattern problem solving, it increases the conjecturing process theories [4] about the type of empirical induction from a finite number of discrete cases which has seven steps and not study the students' thinking process in constructing the conjecture generalization. The results show that the global conjecturing process occurs at the step of action in which subjects build a conjecture by observing and counting the number of squares complete, at the step of process, the object and scheme were perfectly performed.

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